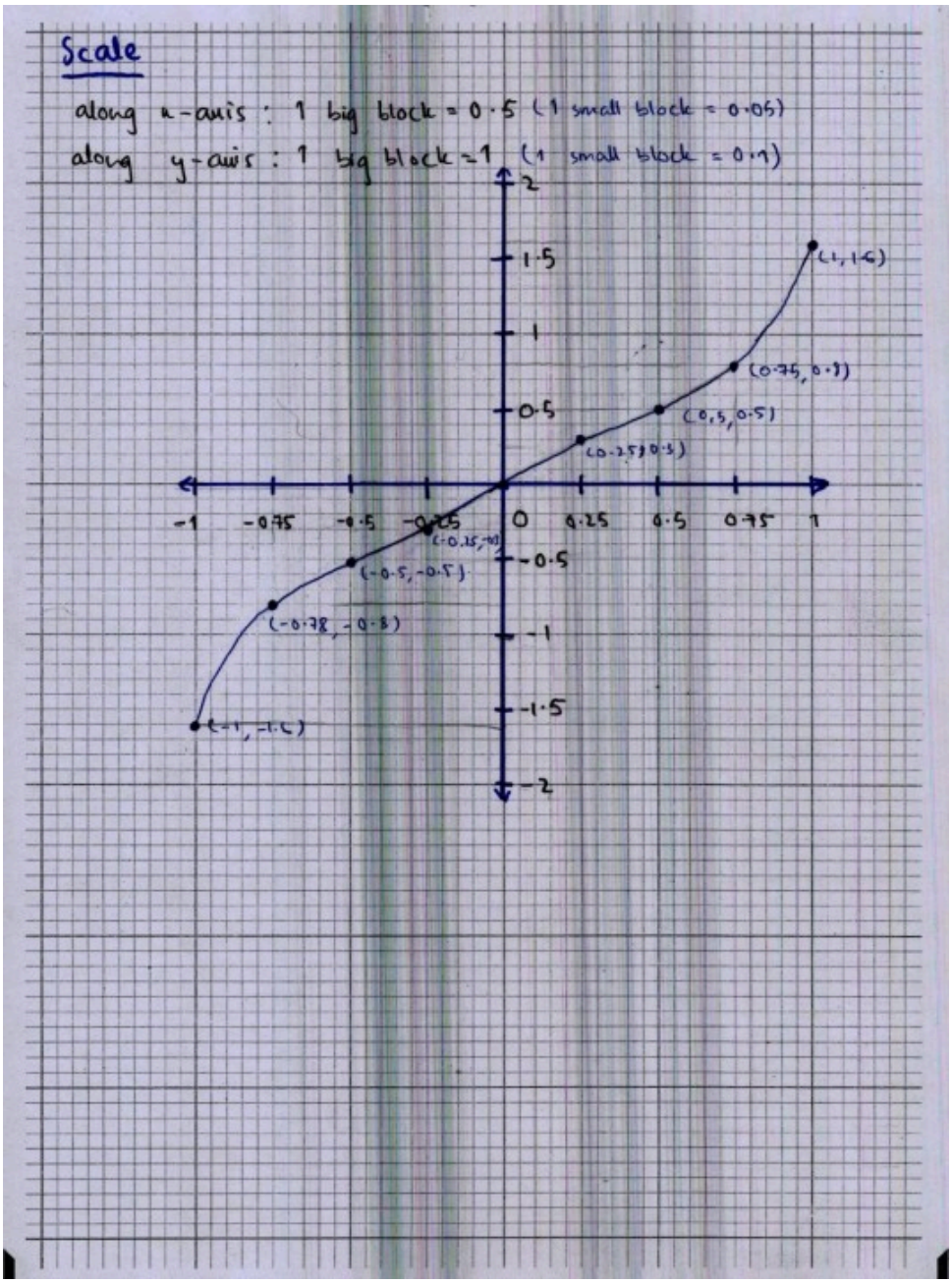
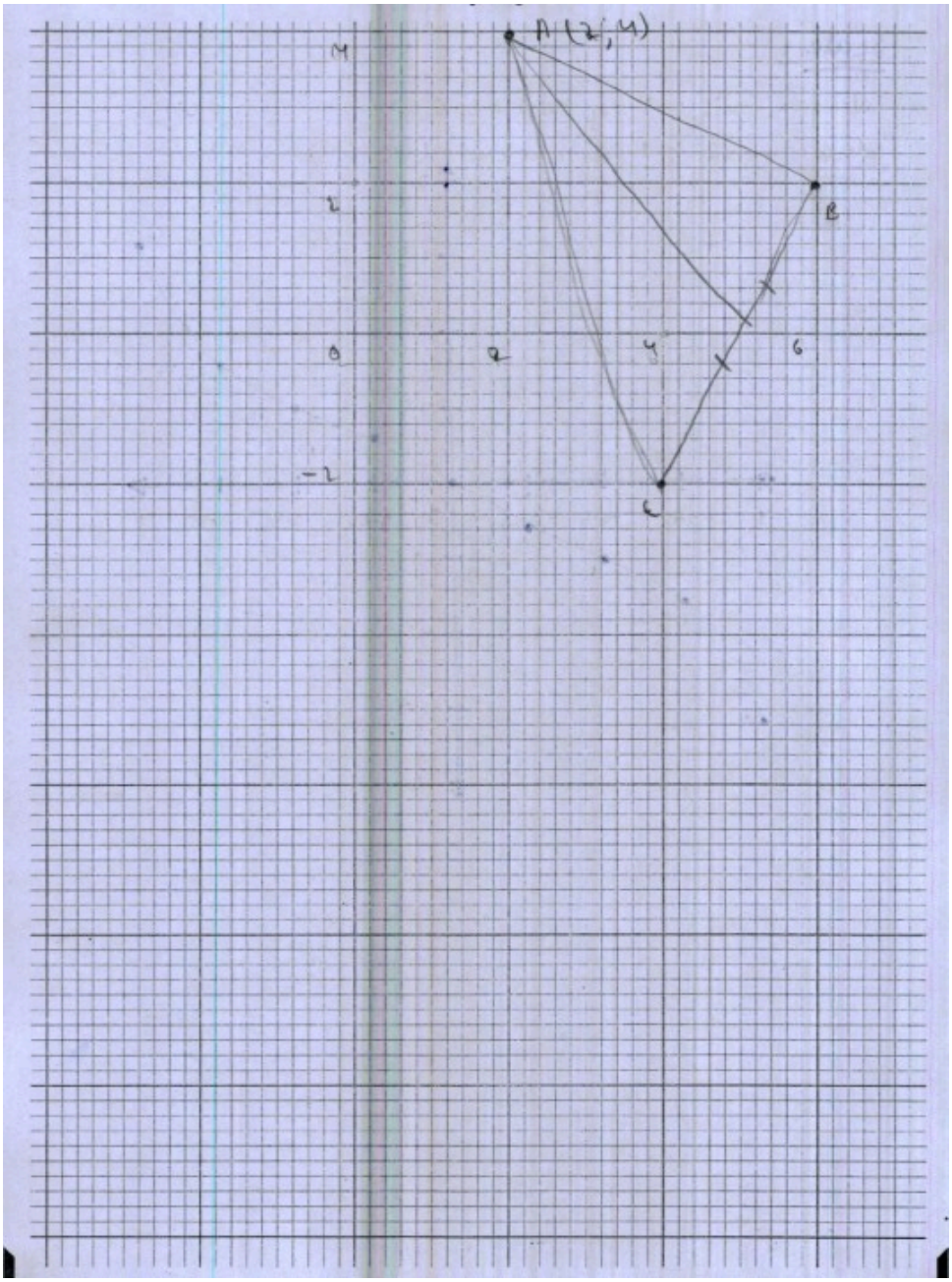
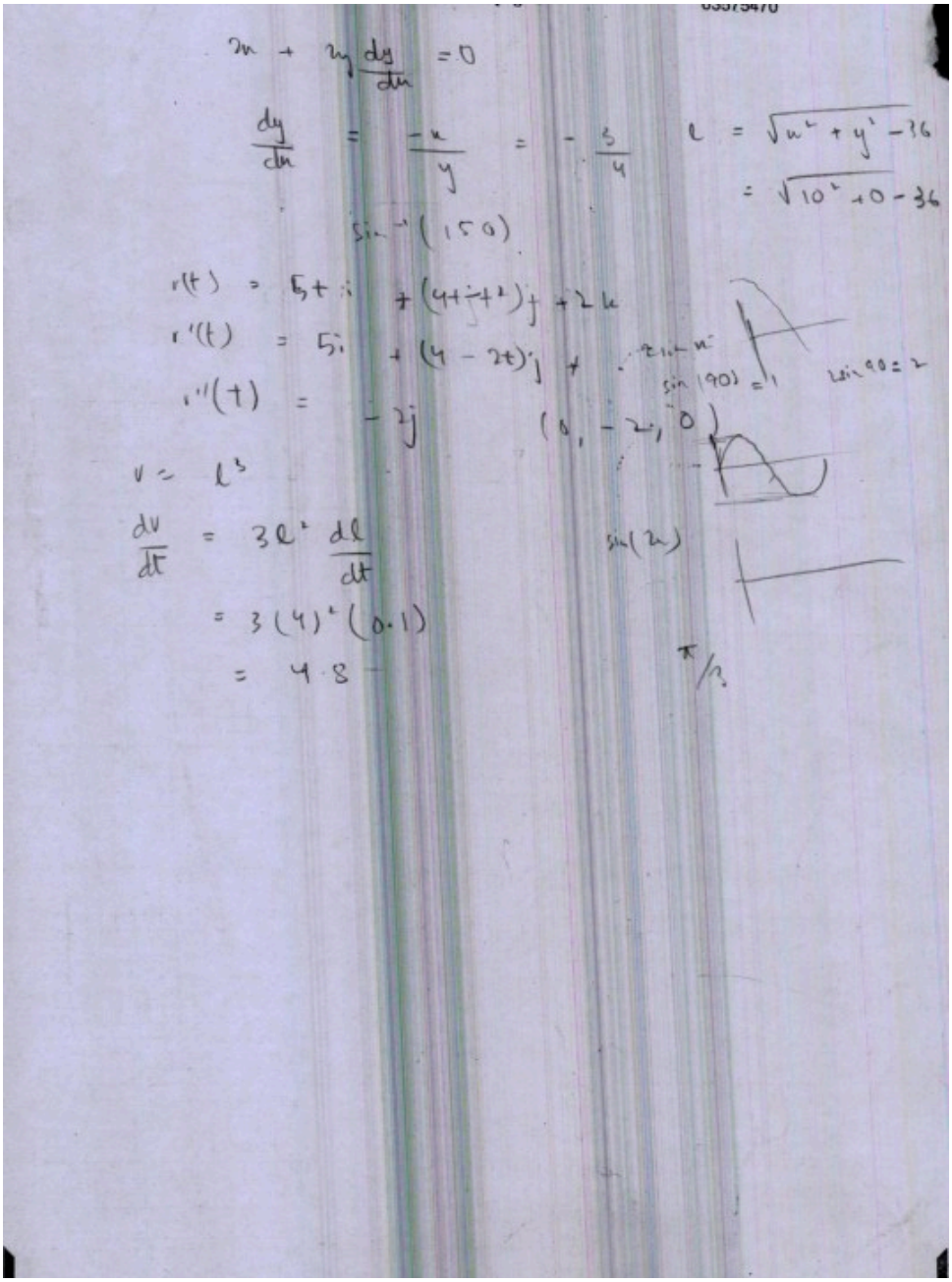

Scale
along x-axis : 1 big block = 0.5 (1 small block = 0.05)
along y-axis : 1 big block = 1 (1 small block = 0.1)
2
1.5
1
0.5
-0.5
-1
-1.5
-2
-1
-0.75
-0.5
-0.25
0
0.25
0.5
0.75
1
(1, 1.6)
(0.75, 0.9)
(0.5, 0.5)
(0.25, 0.3)
(-0.5, -0.5)
(-0.78, -0.8)
(-1, -1.6)


A(2,4)
B
C
4
2
0
-2
2
4
6



$$x + y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x}{y} = -\frac{3}{4}$$

$$l = \sqrt{x^2 + y^2 - 36}$$

$$= \sqrt{10^2 + 0 - 36}$$

$$\sin^{-1}(1.50)$$

$$r(t) = 5t\,i + (4t - t^2)\,j + 2k$$

$$r'(t) = 5i + (4 - 2t)\,j$$

$$r''(t) = -2j \qquad (0, -2, 0)$$

$$V = l^3$$

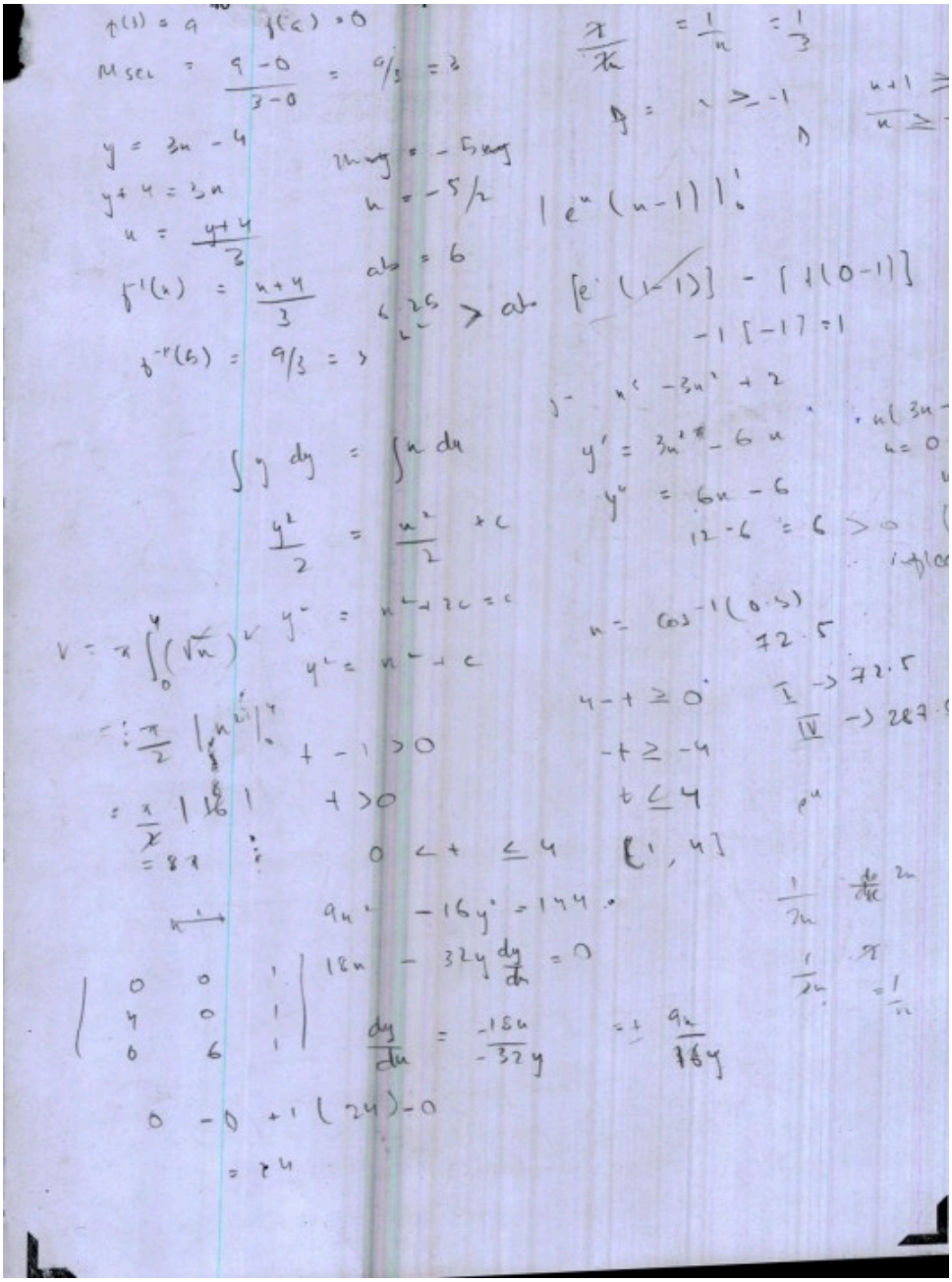
$$\frac{dV}{dt} = 3l^2\frac{dl}{dt}$$

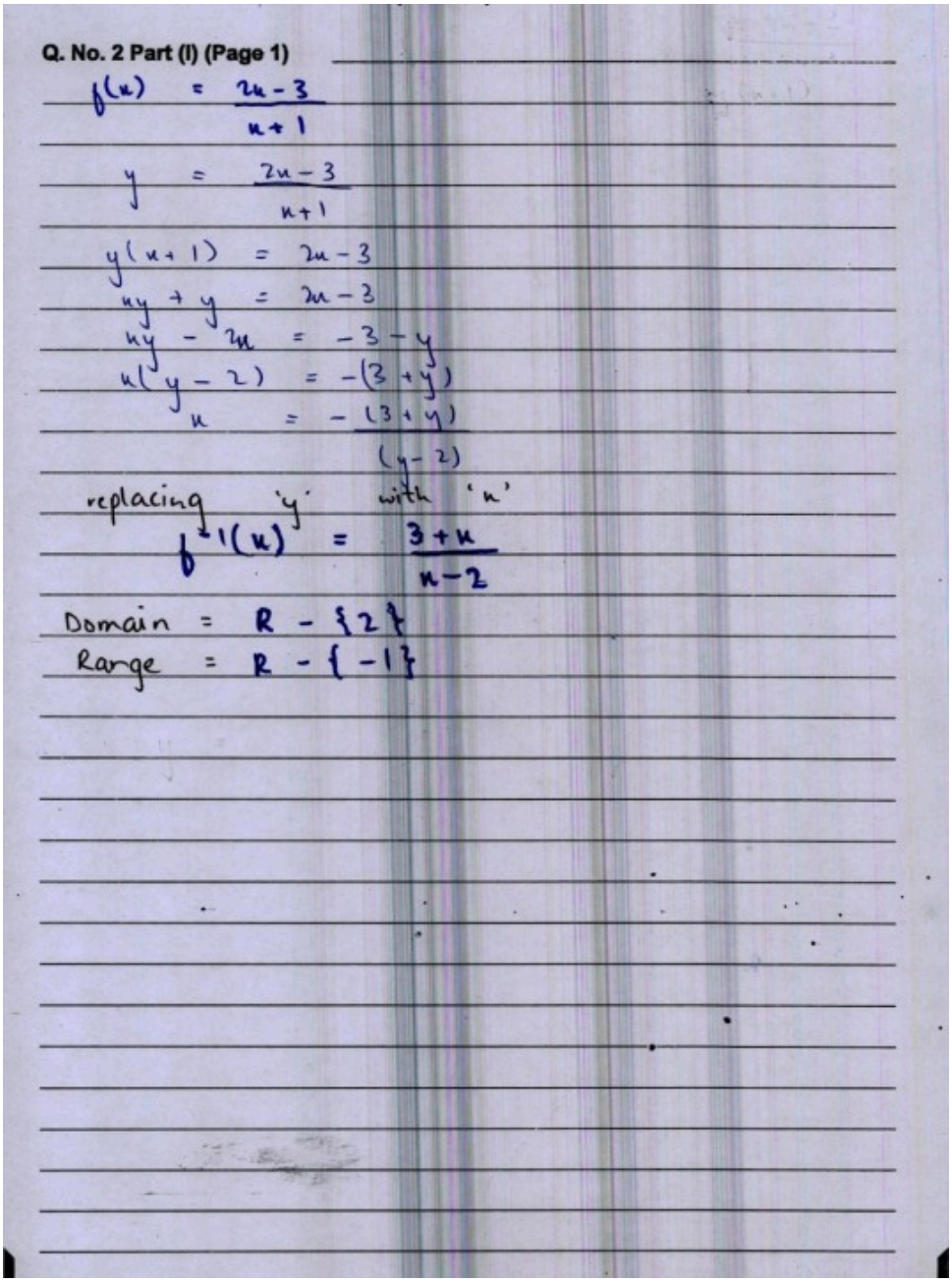
$$= 3(4)^2(0.1)$$

$$= 4.8$$

$$\sin(2x)$$

$$\sin 90 = 1 \qquad \sin^{-1} 90 = 2$$





Q. No. 2 Part (i) (Page 1)

$$f(x) = \frac{2x-3}{x+1}$$

$$y = \frac{2x-3}{x+1}$$

$$y(x+1) = 2x-3$$

$$xy + y = 2x-3$$

$$xy - 2x = -3-y$$

$$x(y-2) = -(3+y)$$

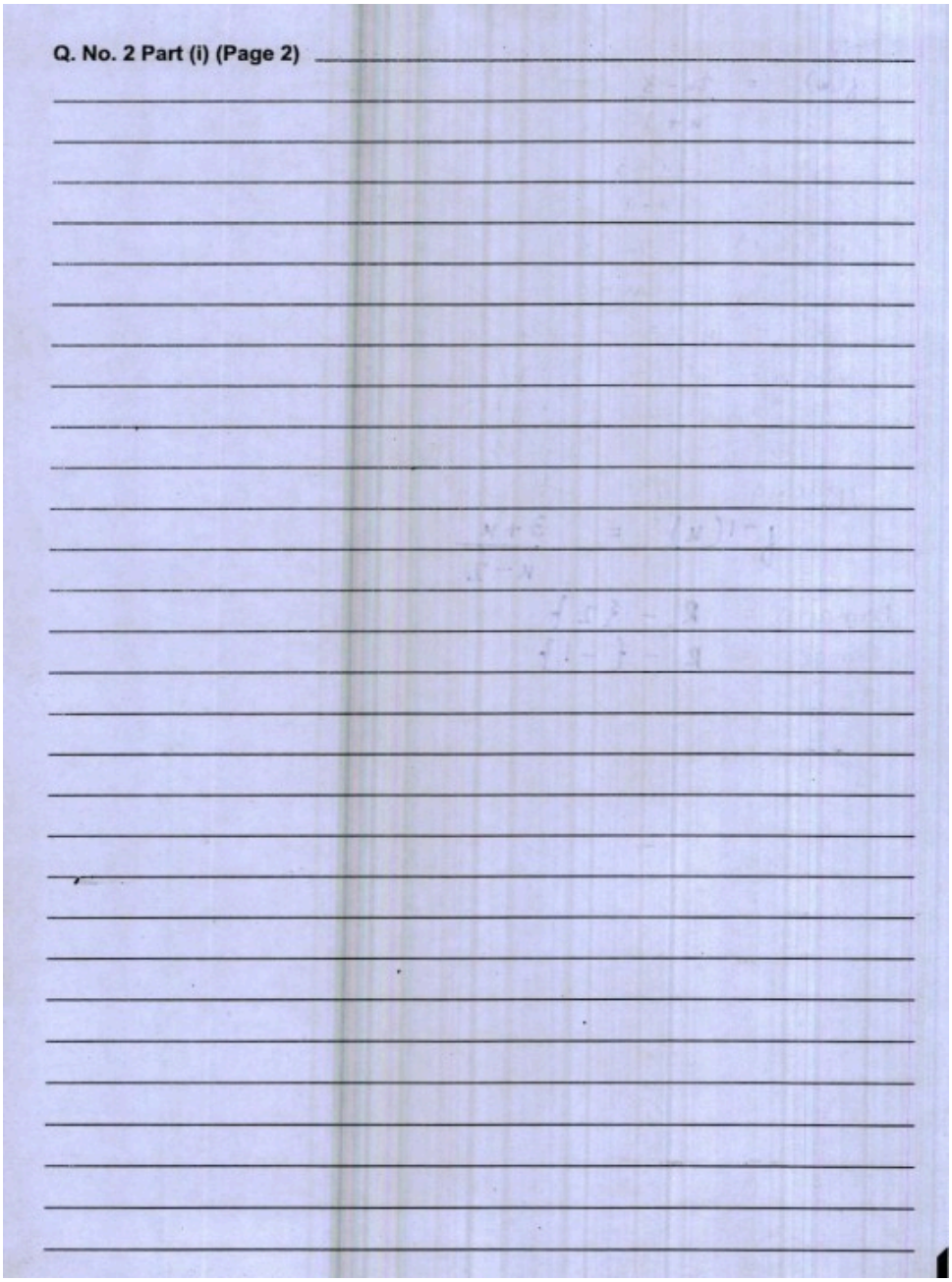
$$x = \frac{-(3+y)}{(y-2)}$$

replacing 'y' with 'x'

$$f^{-1}(x) = \frac{3+x}{x-2}$$

Domain = $R - \{2\}$

Range = $R - \{-1\}$


Q. No. 2 Part (i) (Page 2)

Q. No. 2 Part (II) (Page 1)

$y = e^{2x} \ln(x)$

differentiating w.r.t 'x'

$$\frac{dy}{dx} = e^{2x} \cdot \frac{d}{dx}(2x) \quad [y = e^{2x}]$$

$$= e^{2x} \cdot 2$$

$$\frac{dy}{dx} = 2e^{2x}$$

$$\frac{dy}{dx} = \frac{1}{x} \frac{d}{dx}(x) \quad [y = \ln(x)]$$

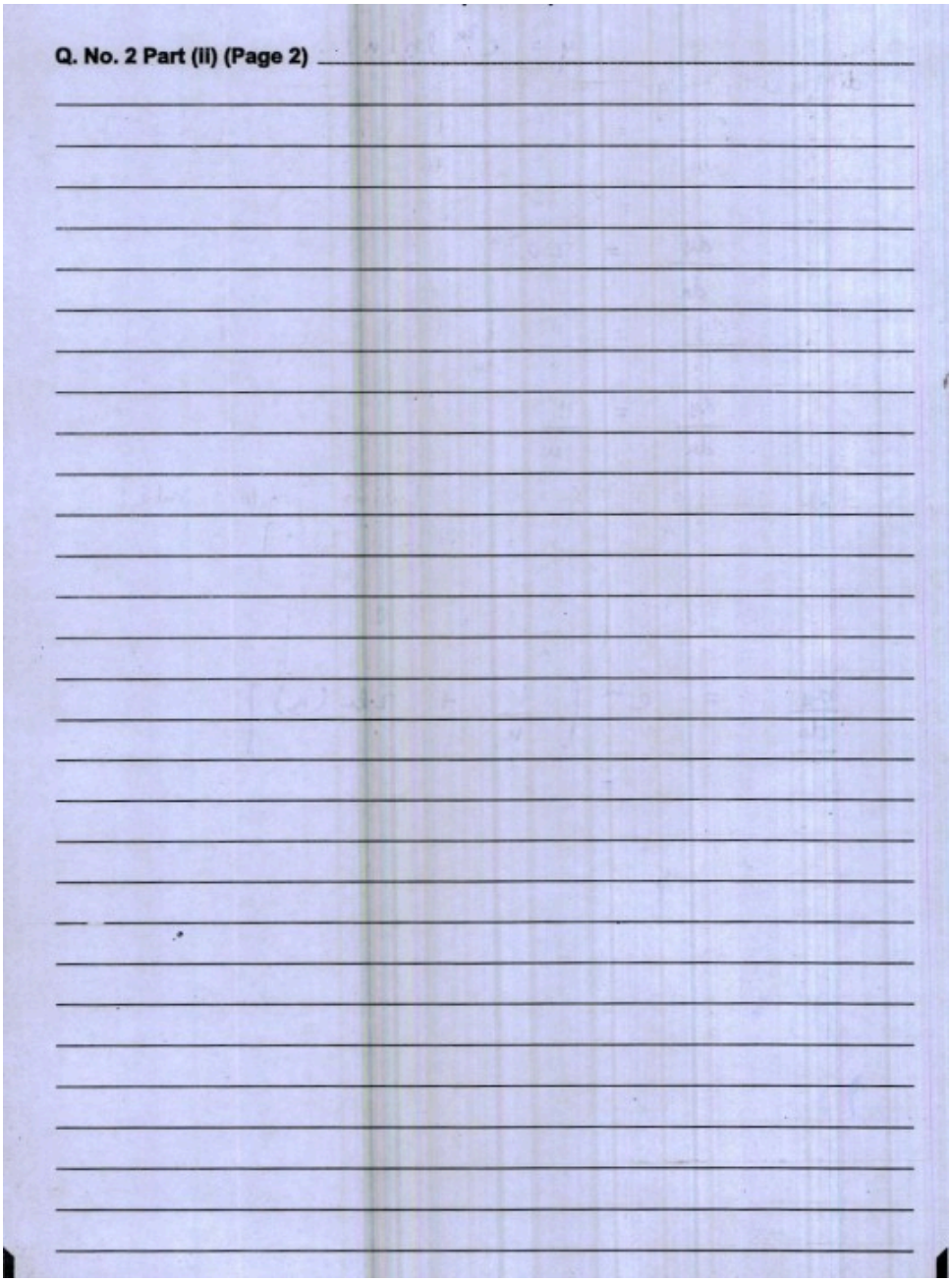
$$\frac{dy}{dx} = \frac{1}{x}$$

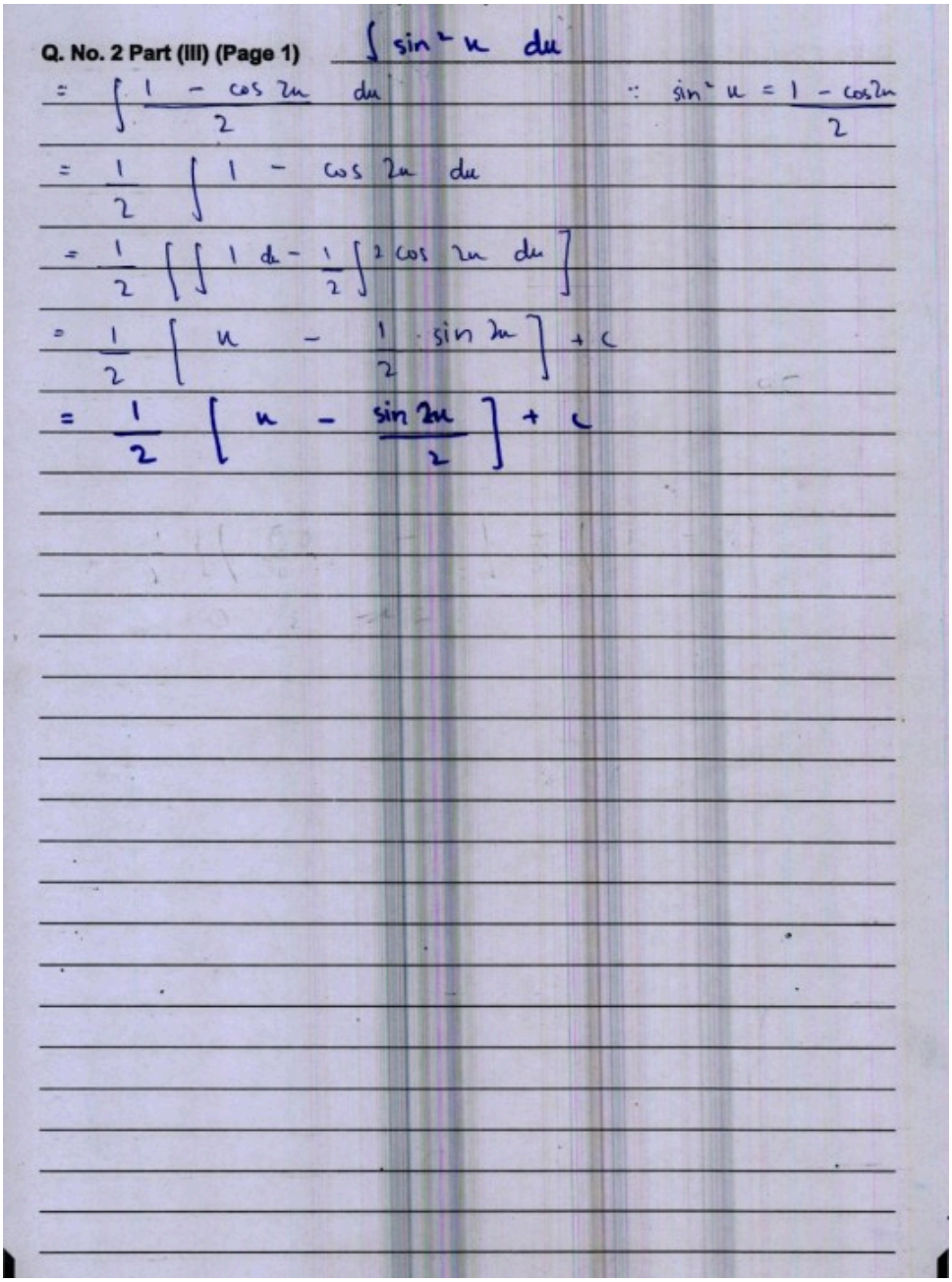
for $y = e^{2x} \ln(x)$ (using product rule)

$$\frac{dy}{dx} = e^{2x}\left(\frac{1}{x}\right) + [\ln(x)]\left(2e^{2x}\right)$$

$$= e^{2x}\left[\frac{1}{x} + 2\ln(x)\right]$$

$$\frac{dy}{dx} = e^{2x}\left[\frac{1}{x} + 2\ln(x)\right]$$


Q. No. 2 Part (ii) (Page 2)



Q. No. 2 Part (III) (Page 1) $\int \sin^2 u \, du$

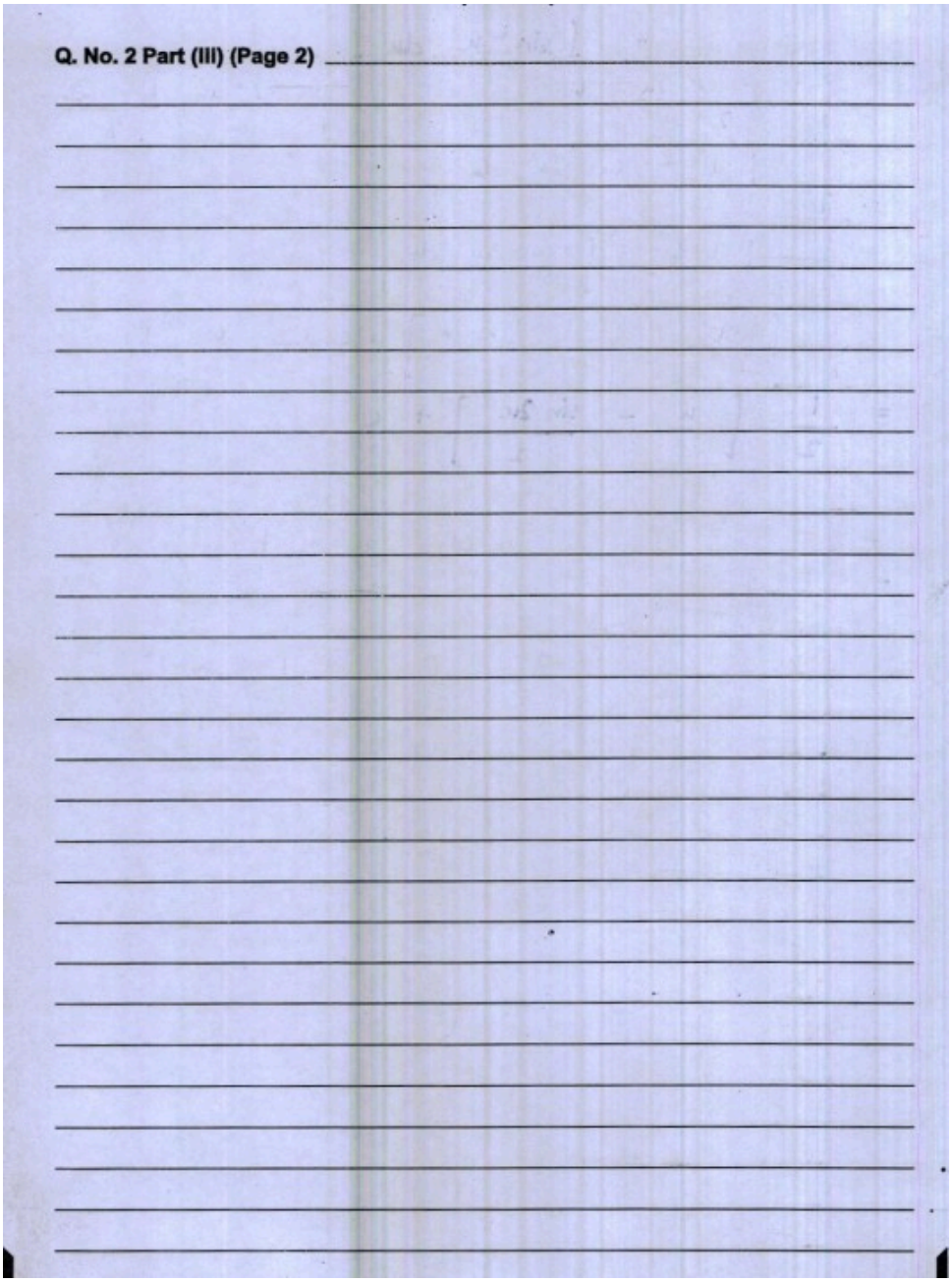
$= \int \frac{1 - \cos 2u}{2} \, du$ $\quad \because \sin^2 u = \frac{1 - \cos 2u}{2}$

$= \frac{1}{2} \int 1 - \cos 2u \, du$

$= \frac{1}{2} \left[\int 1 \, du - \frac{1}{2} \int 2 \cos 2u \, du \right]$

$= \frac{1}{2} \left[u - \frac{1}{2} \sin 2u \right] + c$

$= \frac{1}{2} \left[u - \frac{\sin 2u}{2} \right] + c$


Q. No. 2 Part (III) (Page 2)

Q. No. 2 Part (iv) (Page 1) $\frac{dy}{dx} = \frac{y+1}{x-2}$

$\frac{1}{y+1}\,dy = \frac{1}{x-2}\,dx$

integrating b.s

$\int \frac{1}{y+1}\,dy = \int \frac{1}{x-2}\,dx$

$\ln|y+1| = \ln|x-2| + \ln|c|$

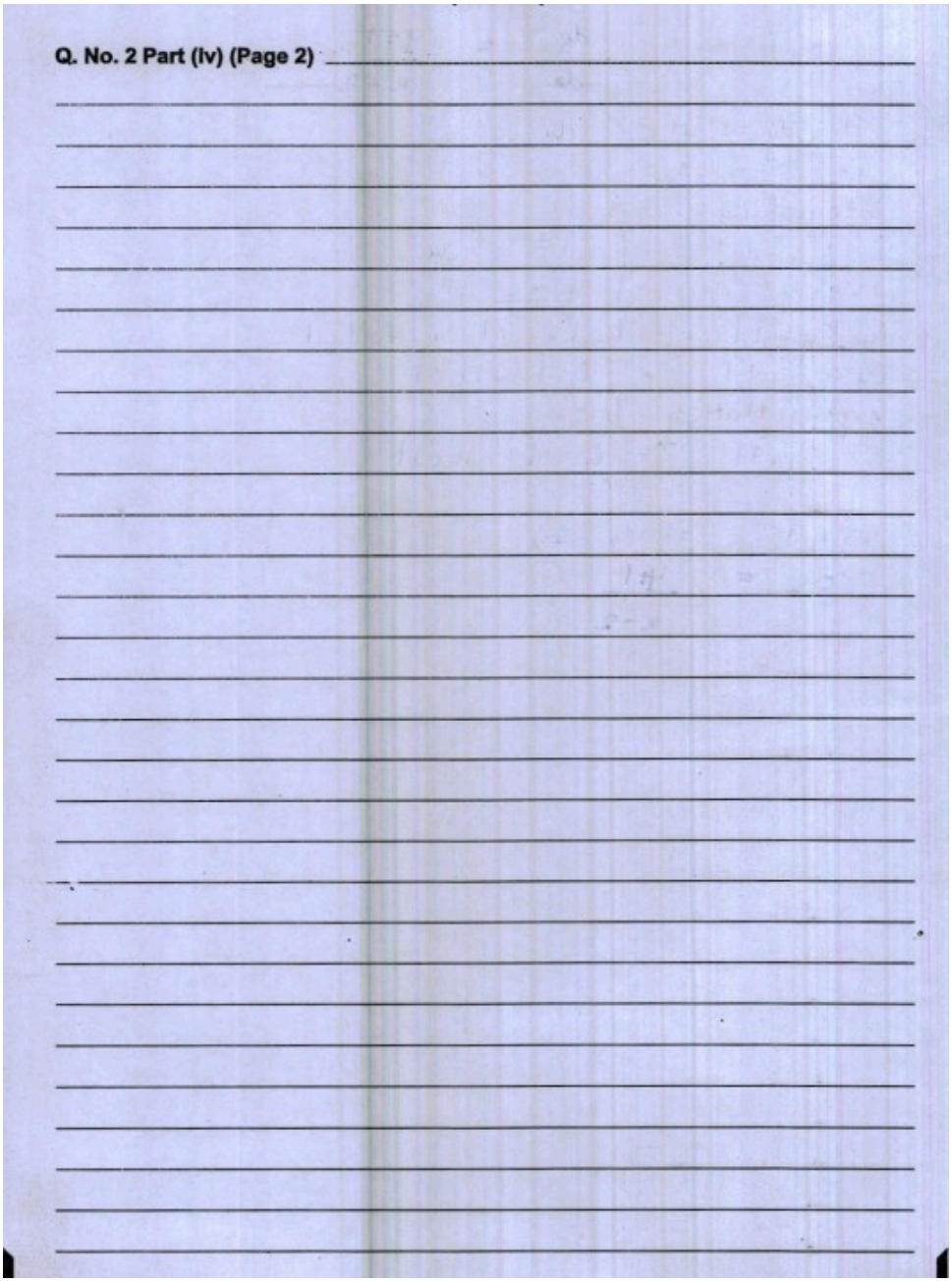
$\ln|y+1| = \ln|x-2||c|$

exponentiating b.s

$e^{\ln|y+1|} = e^{\ln|(x-2)(c)|}$

$y+1 = (x-2)c$

$c = \frac{y+1}{x-2}$


Q. No. 2 Part (iv) (Page 2)

Q. No. 2 Part (v) (Page 1) $3x^2 - 5xy + 2y^2 = 0$

To pass through origin, (0, 0) should satisfy the eq.

$3(0)^2 - 5(0)(0) + 2(0)^2 = 0$

$0 - 0 + 0 = 0$

$0 = 0$

a. ∴ it represents a pair of lines through origin

$3x^2 - 5xy + 2y^2 = 0$

$3x^2 - 3xy - 2xy + 2y^2 = 0$

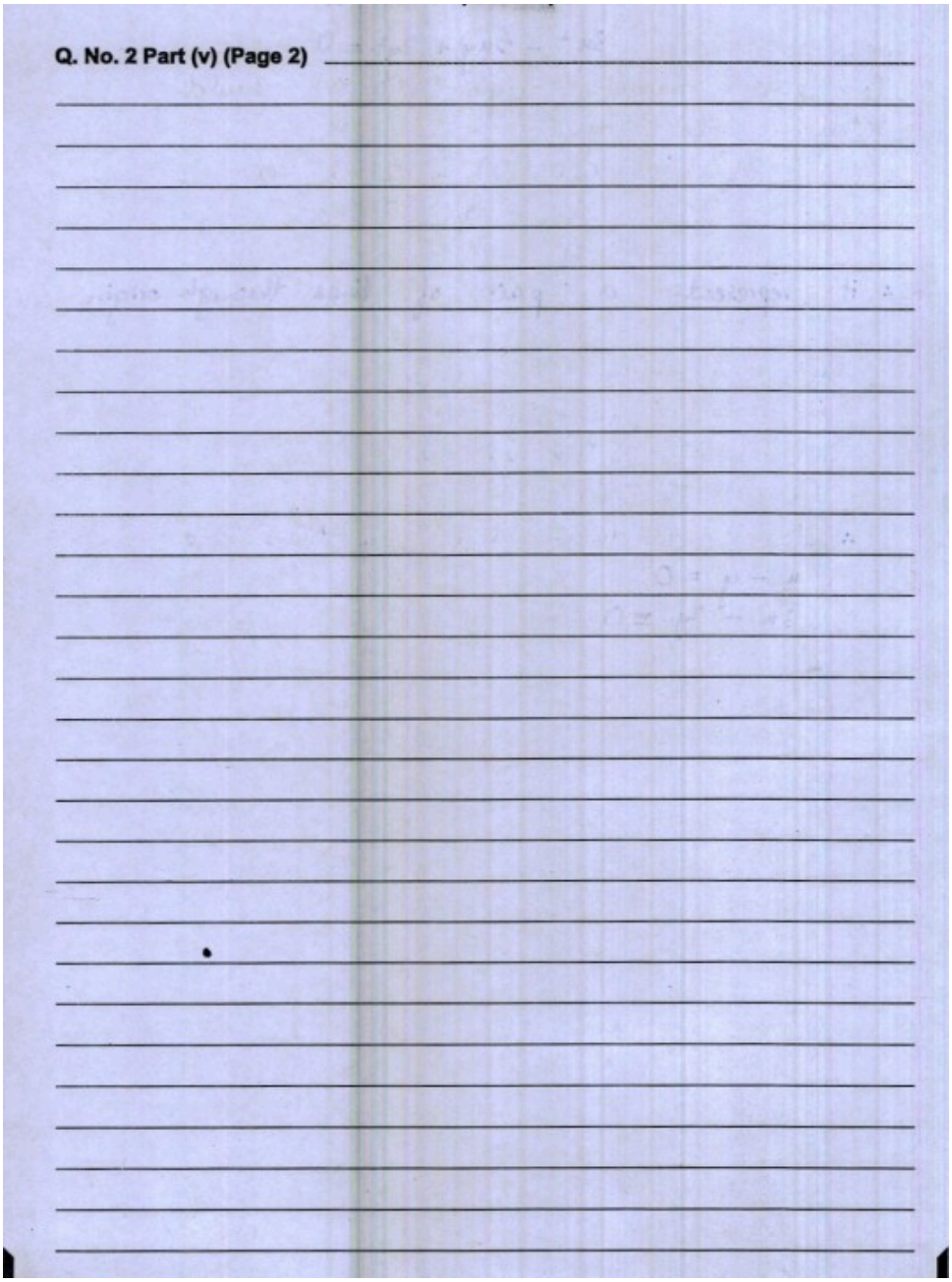
$3x(x - y) - 2y(x - y) = 0$

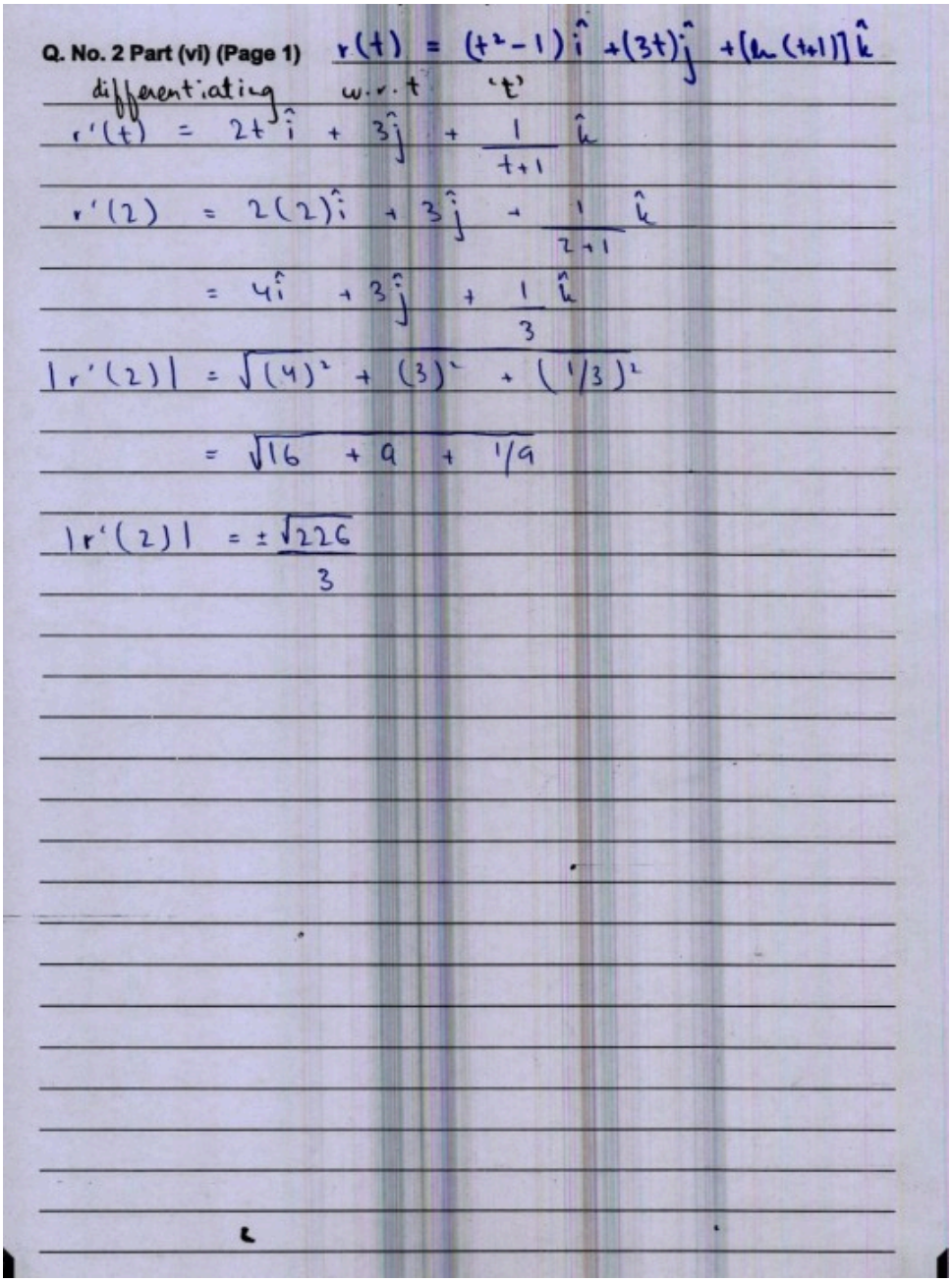
$(x - y)(3x - 2y) = 0$

∴ the individual line equations are:

b. $x - y = 0$

$3x - 2y = 0$


Q. No. 2 Part (v) (Page 2)



Q. No. 2 Part (vi) (Page 1) $r(t) = (t^2-1)\hat{i} + (3t)\hat{j} + [\ln(t+1)]\hat{k}$

differentiating w.r.t 't'

$$r'(t) = 2t\hat{i} + 3\hat{j} + \frac{1}{t+1}\hat{k}$$

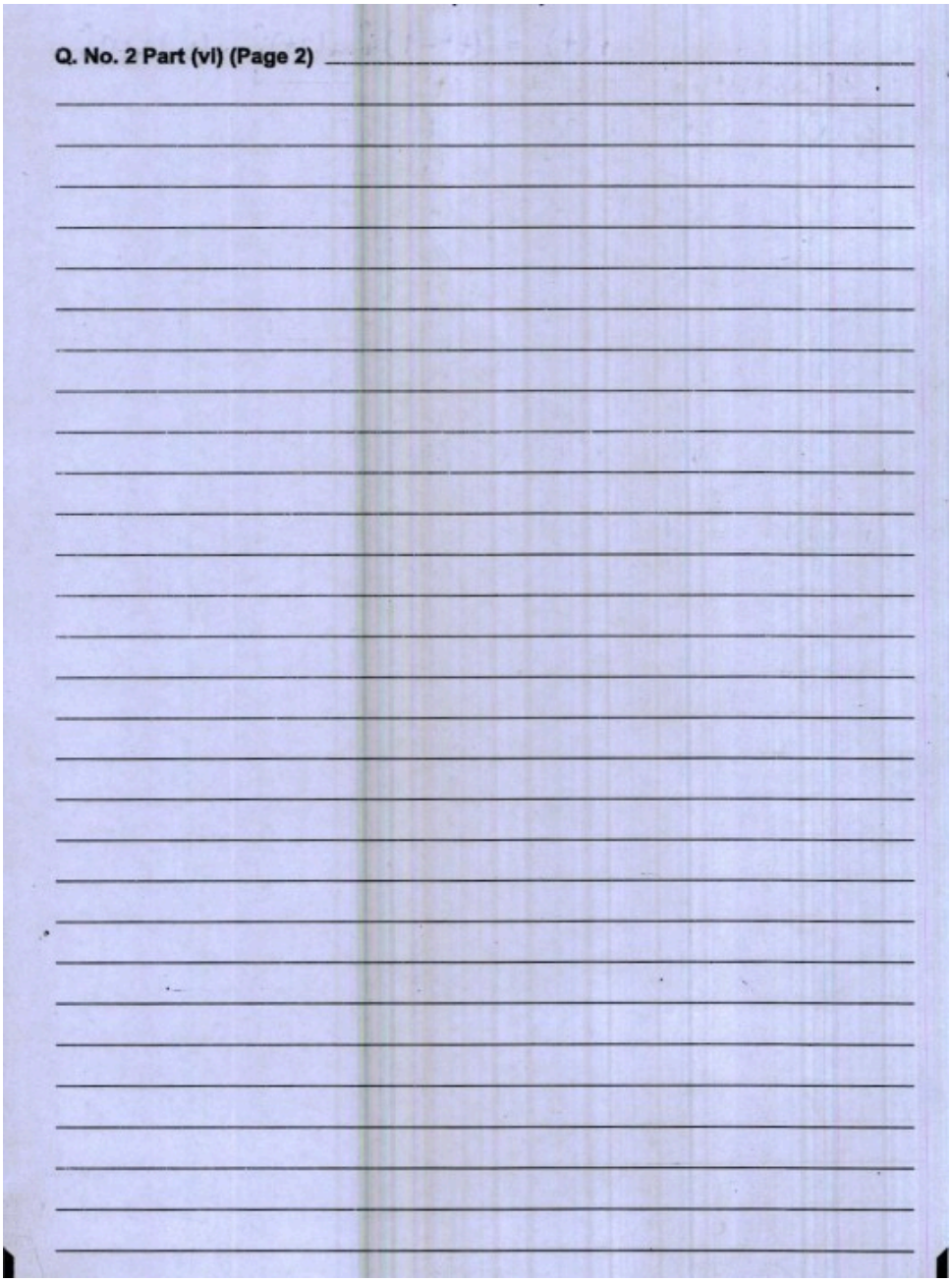
$$r'(2) = 2(2)\hat{i} + 3\hat{j} + \frac{1}{2+1}\hat{k}$$

$$= 4\hat{i} + 3\hat{j} + \frac{1}{3}\hat{k}$$

$$|r'(2)| = \sqrt{(4)^2 + (3)^2 + (1/3)^2}$$

$$= \sqrt{16 + 9 + 1/9}$$

$$|r'(2)| = \pm\frac{\sqrt{226}}{3}$$


Q. No. 2 Part (vi) (Page 2)

Q. No. 2 Part (vii) (Page 1)

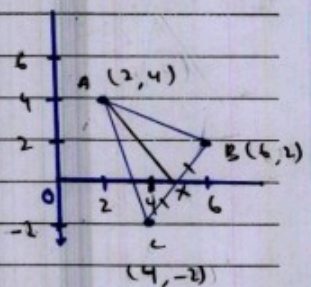
Let X be the midpoint of BC

X = midpoint BC

$= \left(\frac{6+4}{2}, \frac{2-2}{2}\right)$

$= \left(\frac{10}{2}, \frac{0}{2}\right)$

$= (5, 0)$



Let slope of AX = m

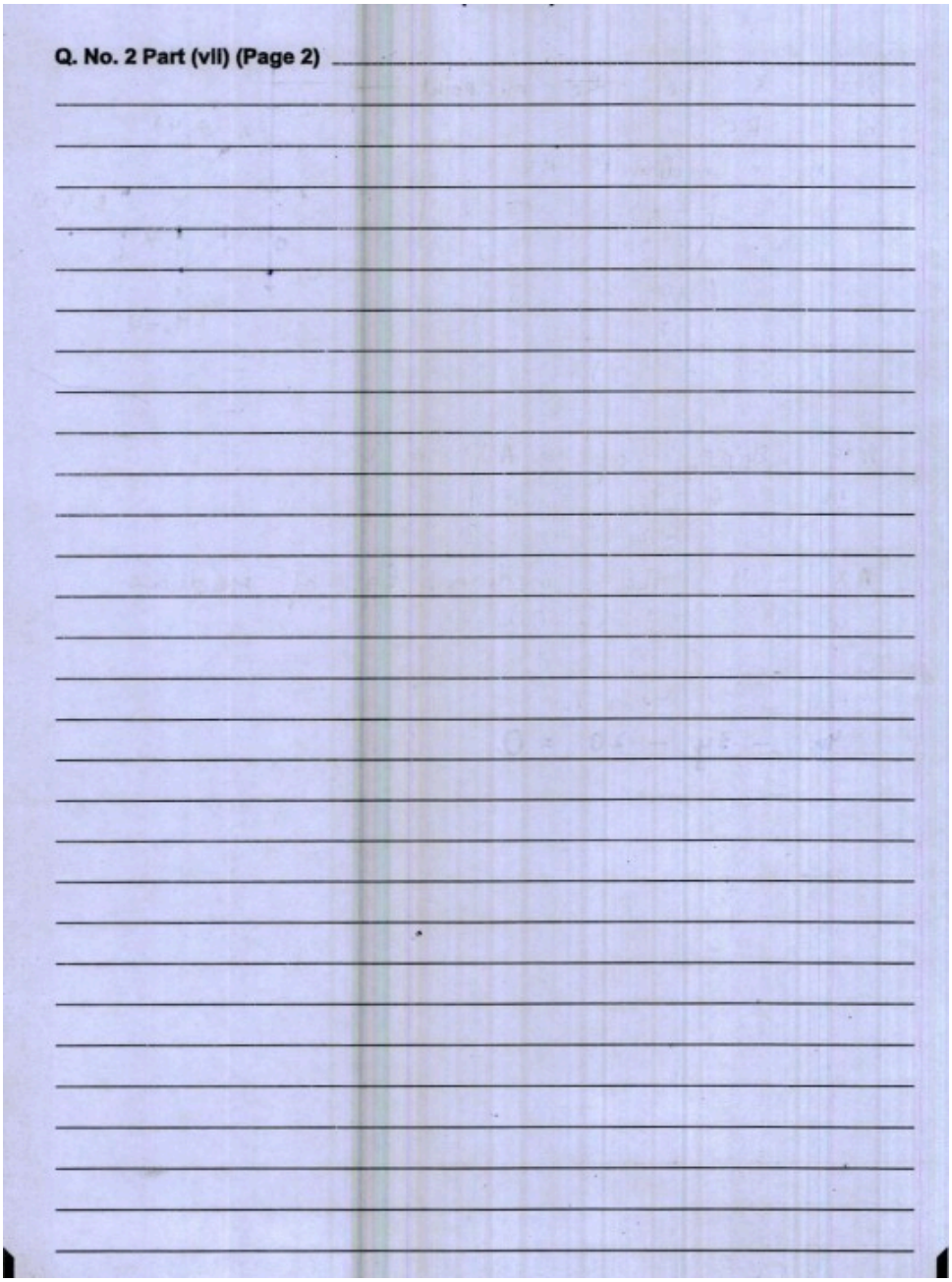
$m = \frac{0-4}{5-2} = \frac{-4}{3}$

AX is the median. Eq. of Median

$y - 0 = \frac{-4}{3}(x - 5)$

$3y = -4x + 20$

$4x - 3y - 20 = 0$


Q. No. 2 Part (vii) (Page 2)

Q. No. 2 Part (viii) (Page 1)

Given :-

$r = 10$ cm

$\frac{dr}{dx} = 0.05$ cm

To Find :-

Approximate %age error in V = ?

Solution :-

$V = \frac{4}{3}\pi r^3$

differentiating w.r.t. 'x'

$\frac{dV}{dx} = \left(\frac{4}{3}\pi\right)\left(3r^2 \frac{dr}{dx}\right)$ $\quad [V = 4188.8\ cm^3]$

$= 4\pi r^2 \frac{dr}{dx}$

$= 4\pi (10)^2 (0.05)$

$\frac{dV}{dx} = 62.8\ cm^3 = 0.015\ cm^3$

$\frac{dV}{dx} = \frac{62.8}{4188.8} \times 100\ \%$.

PTO

Q. No. 2 Part (viii) (Page 2) $f(x) = x^3 - 2x - 5 \quad [2, 3]$

$x_0 = 2 \qquad x_1 = 3$

$f(x_0) = -1 < 0 \qquad f(x_1) = 16 > 0$

atleast 1 sol. exists.

First iteration

$x_2 = \frac{2+3}{2} = 2.5$

$f(x_2) = 5.6 > 0$

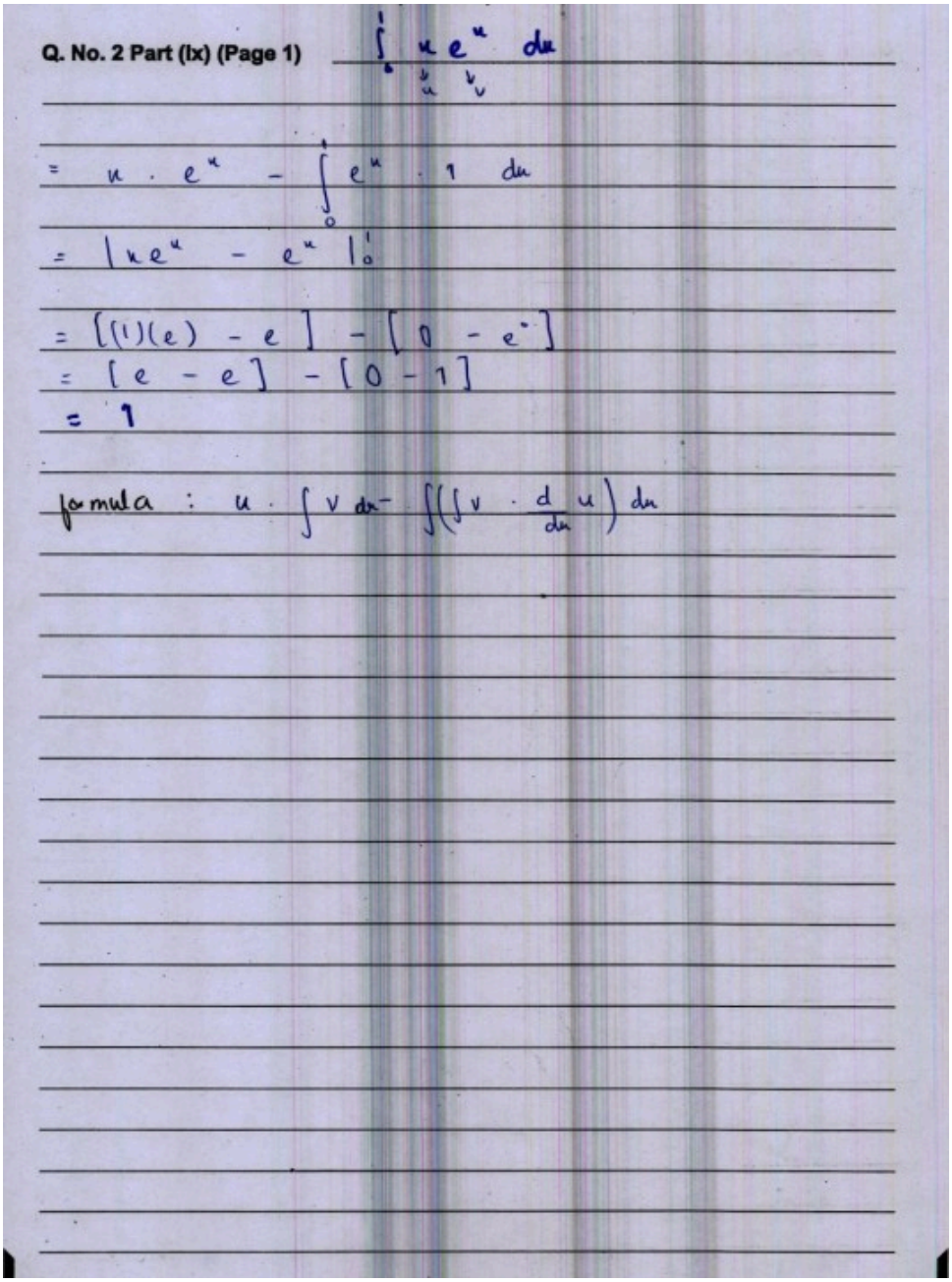
$[2, 2.5]$

second iteration

$x_3 = \frac{2+2.5}{2} = 2.25$

$f(x_3) = 1.89 > 0$

$[2, 2.25]$



Q. No. 2 Part (lx) (Page 1) $\int_0^1 x e^x \, dx$

$u \quad v$

$= x \cdot e^x - \int_0^1 e^x \cdot 1 \, dx$

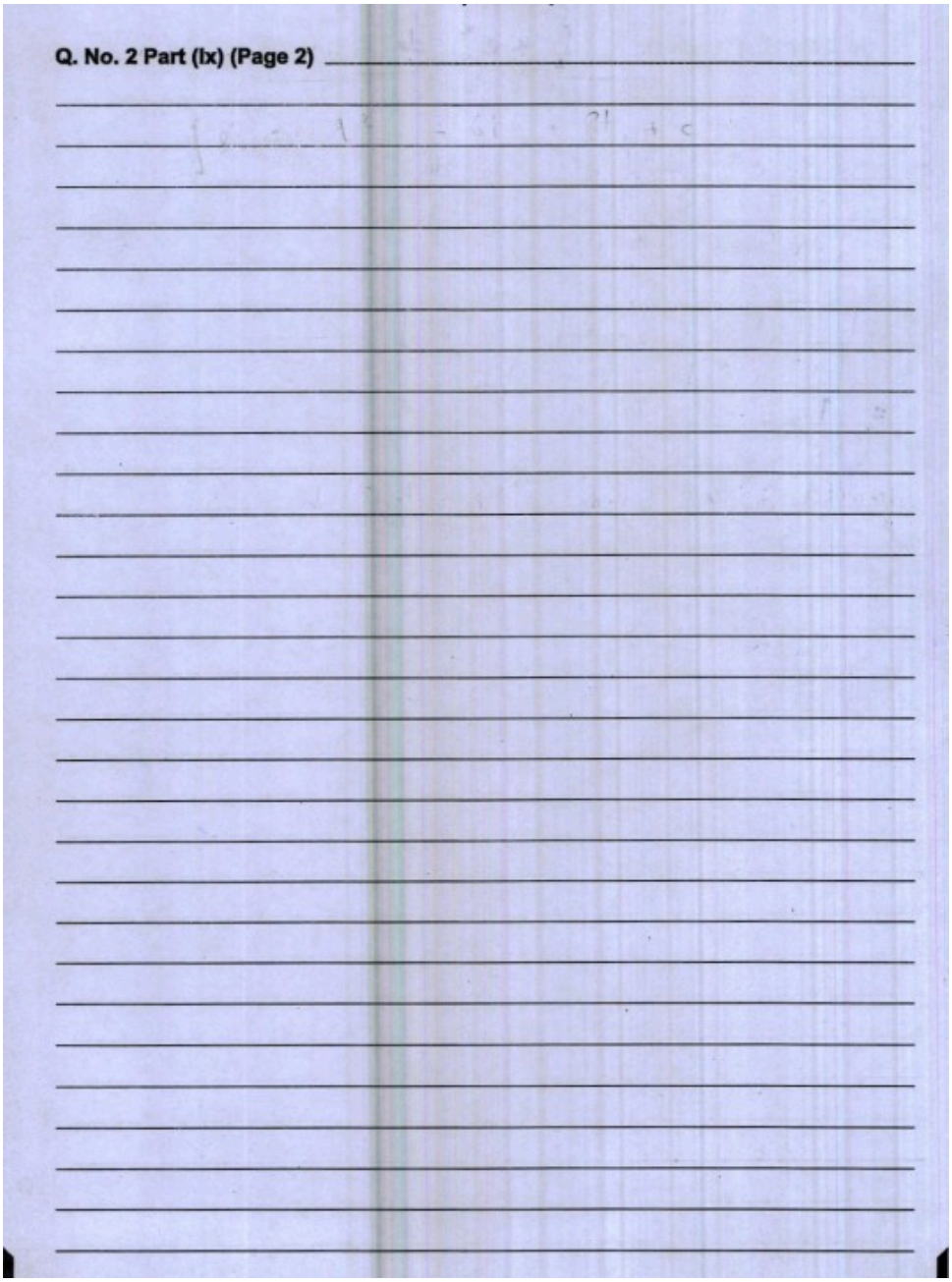
$= |xe^x - e^x|_0^1$

$= [(1)(e) - e] - [0 - e^0]$

$= [e - e] - [0 - 1]$

$= 1$

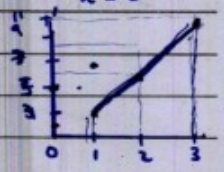
formula : $u \cdot \int v \, dx - \int \left(\int v \cdot \frac{d}{dx} u \right) dx$


Q. No. 2 Part (ix) (Page 2)

Q. No. 2 Part (x) (Page 1)

$y = x^2 + 2$ $\quad x = 1 \quad x = 3$

y — axis



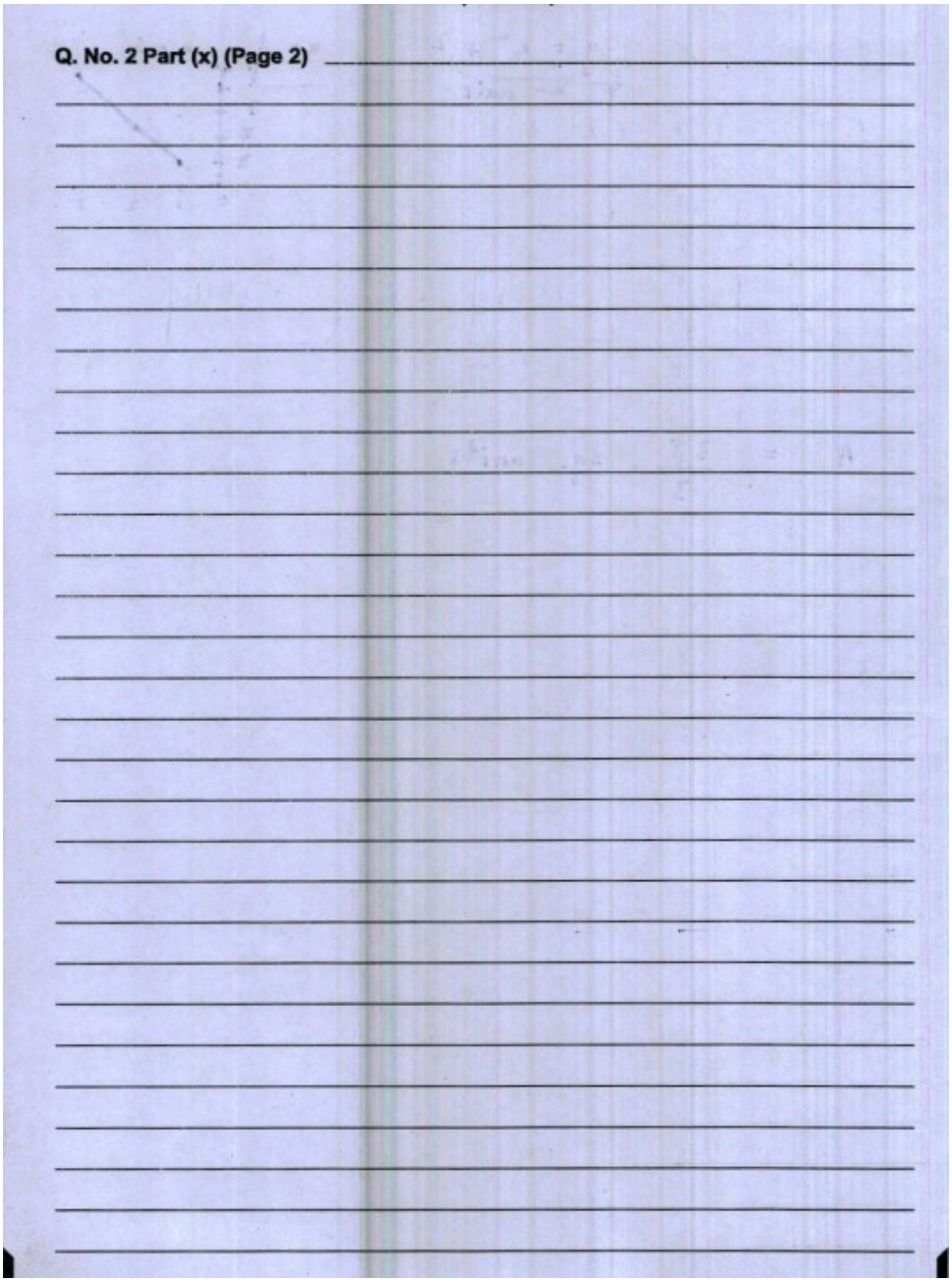
$$A = \int_1^3 x^2 + 2 \, dx$$

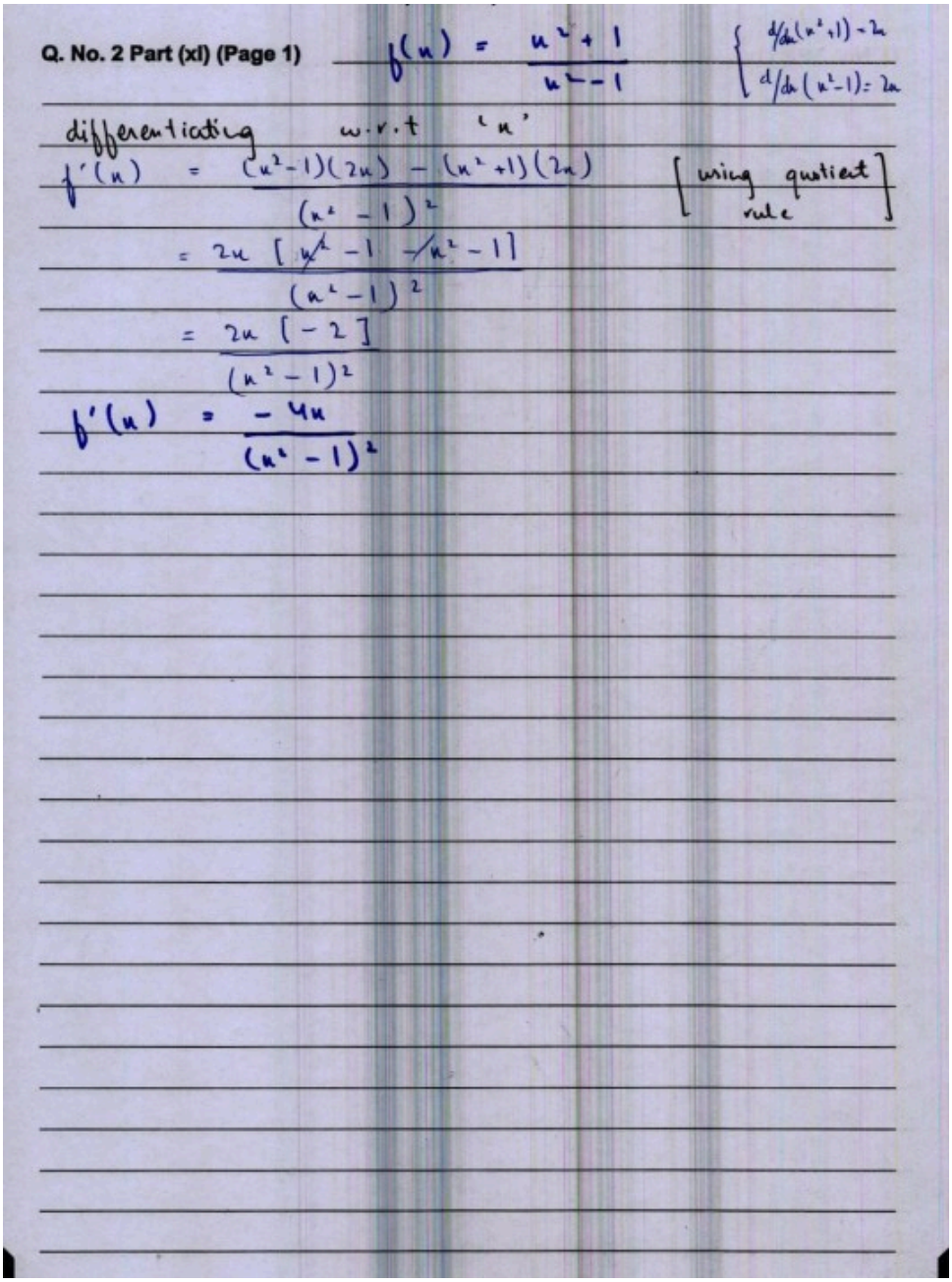
$$= \left| \frac{x^3}{3} + 2x \right|_1^3$$

$$= \left[\frac{3^3}{3} + 2(3) \right] - \left[\frac{1^3}{3} + 2(1) \right]$$

$$= \left[15 \right] - \left[\frac{7}{3} \right]$$

$$A = \frac{38}{3} \text{ sq. units}$$


Q. No. 2 Part (x) (Page 2)



Q. No. 2 Part (xl) (Page 1)

$f(u) = \frac{u^2+1}{u^2-1}$ $\left\{ \frac{d}{du}(u^2+1) = 2u \right.$ $\left. \frac{d}{du}(u^2-1) = 2u \right\}$

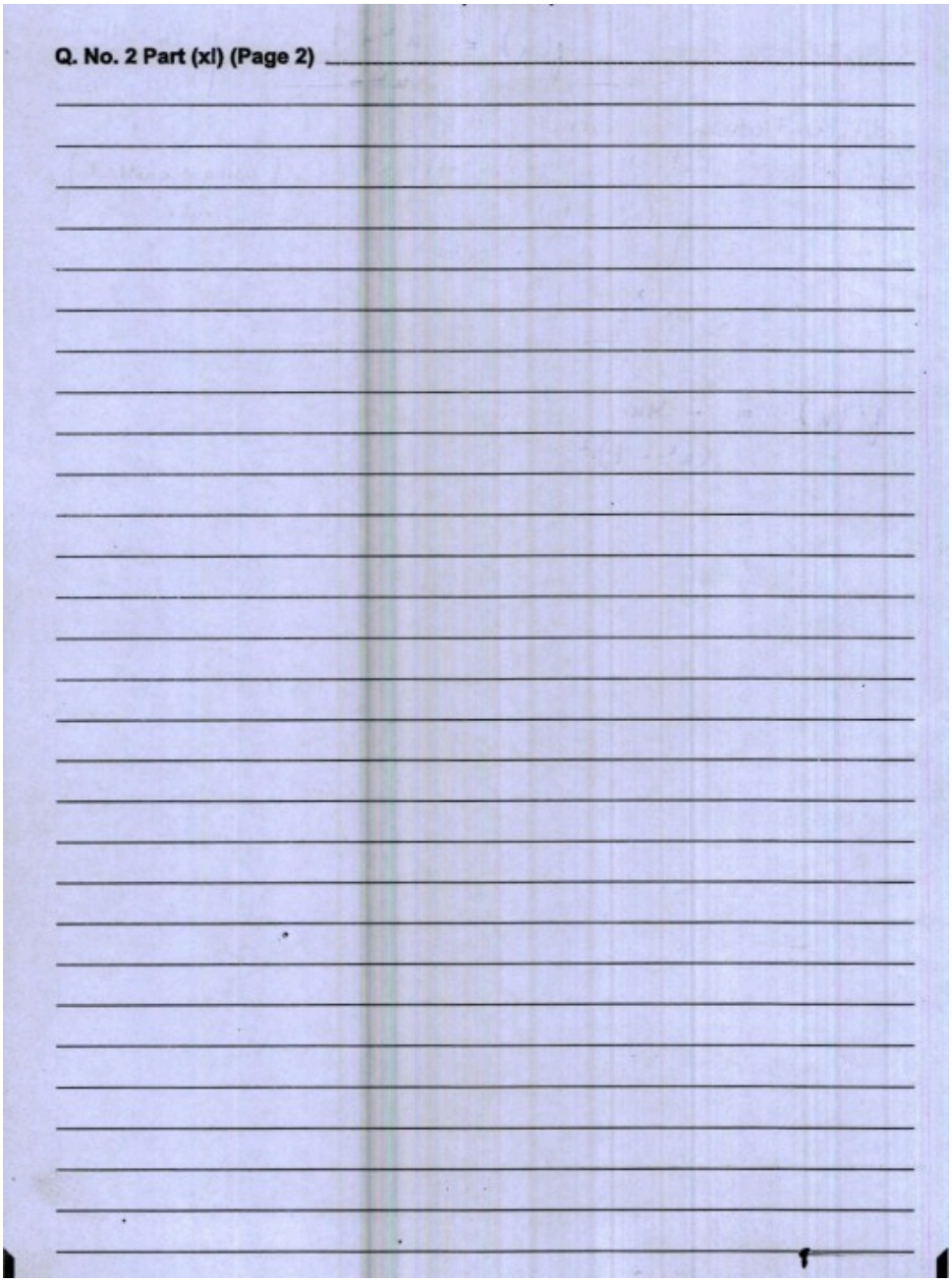
differentiating w.r.t 'u'

$f'(u) = \frac{(u^2-1)(2u) - (u^2+1)(2u)}{(u^2-1)^2}$ [using quotient rule]

$= \frac{2u[u^2 - 1 - u^2 - 1]}{(u^2-1)^2}$

$= \frac{2u[-2]}{(u^2-1)^2}$

$f'(u) = \frac{-4u}{(u^2-1)^2}$


Q. No. 2 Part (xl) (Page 2)

Q. No. 2 Part (xii) (Page 1)

$f(x) = \sqrt{x-1}$

$y = \sqrt{x-1}$

squaring b.s

$y^2 = (\sqrt{x-1})^2$

$y^2 = x - 1$

$x = y^2 + 1$

replace y with x

$f^{-1}(x) = x^2 + 1$

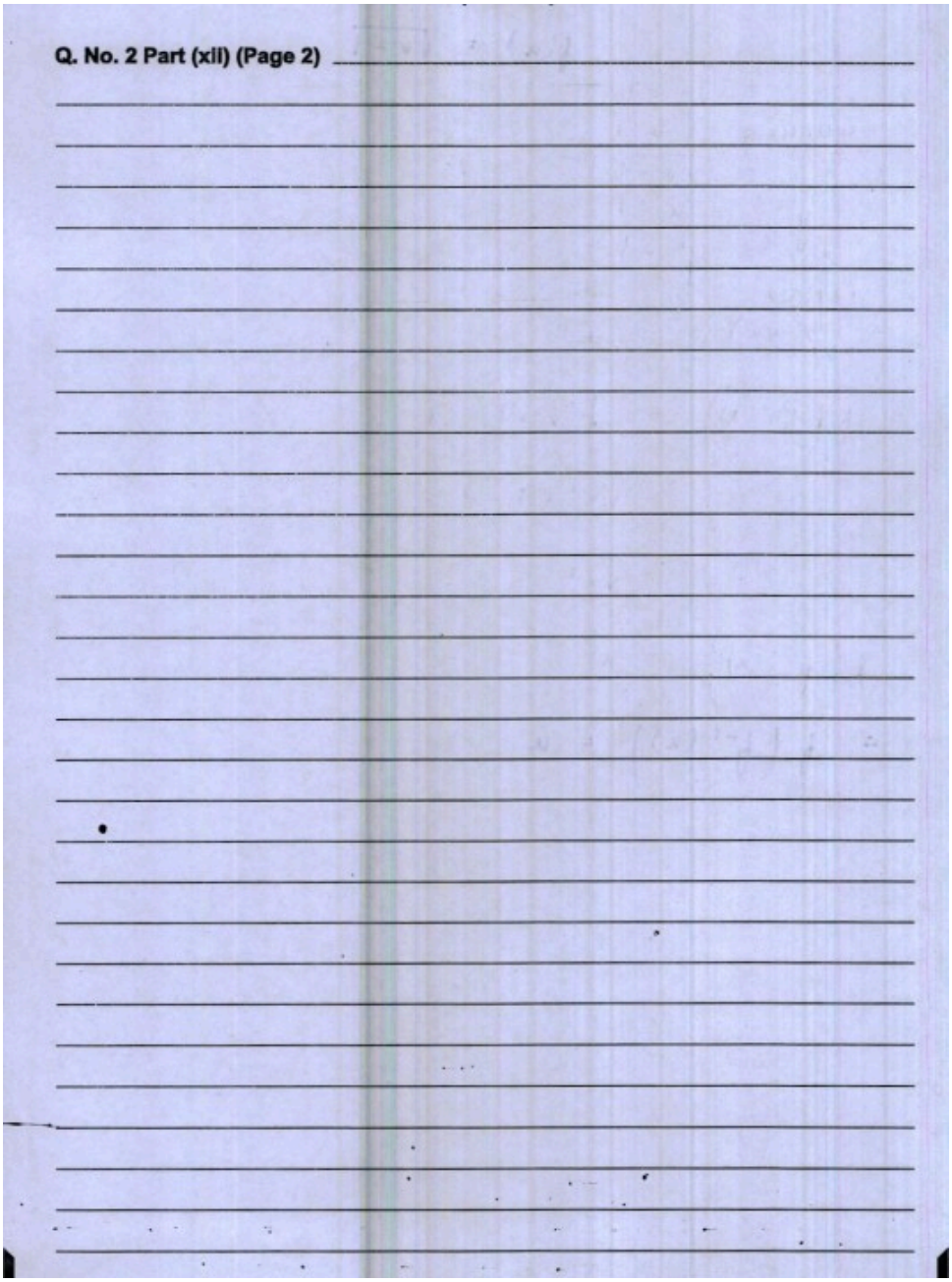
$f(f^{-1}(x)) = \sqrt{(x^2+1) - 1}$

$= \sqrt{x^2 + 1 - 1}$

$= \sqrt{x^2}$

$f(f^{-1}(x)) = x$

$\therefore f(f^{-1}(x)) = x$


Q. No. 2 Part (xii) (Page 2)

(Section C)

Q. No. 3 (Page 1) (0, 0) (2, 0) (0, 2)

eq. of circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \text{—(1)}$$

as the circle passes through these points, they should satisfy the eq.

(0, 0)

$$0^2 + 0^2 + 2g(0) + 2f(0) + c = 0$$

$$c = 0$$

(2, 0)

$$2^2 + 0^2 + 2g(2) + 2f(0) + 0 = 0$$

$$4 + 4g = 0$$

$$4g = -4$$

$$g = -4/4$$

$$g = -1$$

(0, 2)

$$0^2 + 2^2 + 2g(0) + 2f(2) + 0 = 0$$

$$4 + 4f = 0$$

$$4f = -4$$

$$f = -4/4$$

$$f = -1$$

Put values in (1)

$$x^2 + y^2 + 2(-1)x + 2(-1)fy + (0) = 0$$

a. $$x^2 + y^2 - 2x - 2y = 0$$

differentiating w.r.t 'x'

$$2x + 2y\frac{dy}{dx} - 2 - 2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2y - 2) = 2 - 2x$$

Q. No. 3 (Page 2)

$$\frac{dy}{dx} = \frac{2 - 2x}{2y - 2}$$

$$\frac{dy}{dx} = \frac{1 - x}{y - 1}$$

$$M_{tan} = \left.\frac{dy}{dx}\right|_{(0,2)} = \frac{1 - 0}{2 - 1} = \frac{1}{1} = 1$$

b. eq. of tan at $(0, 2)$

$$y - 2 = M_{tan}(x - 0)$$

$$y - 2 = 1(x)$$

$$\therefore x - y + 2 = 0$$

c. eq. of normal at $(0, 2)$

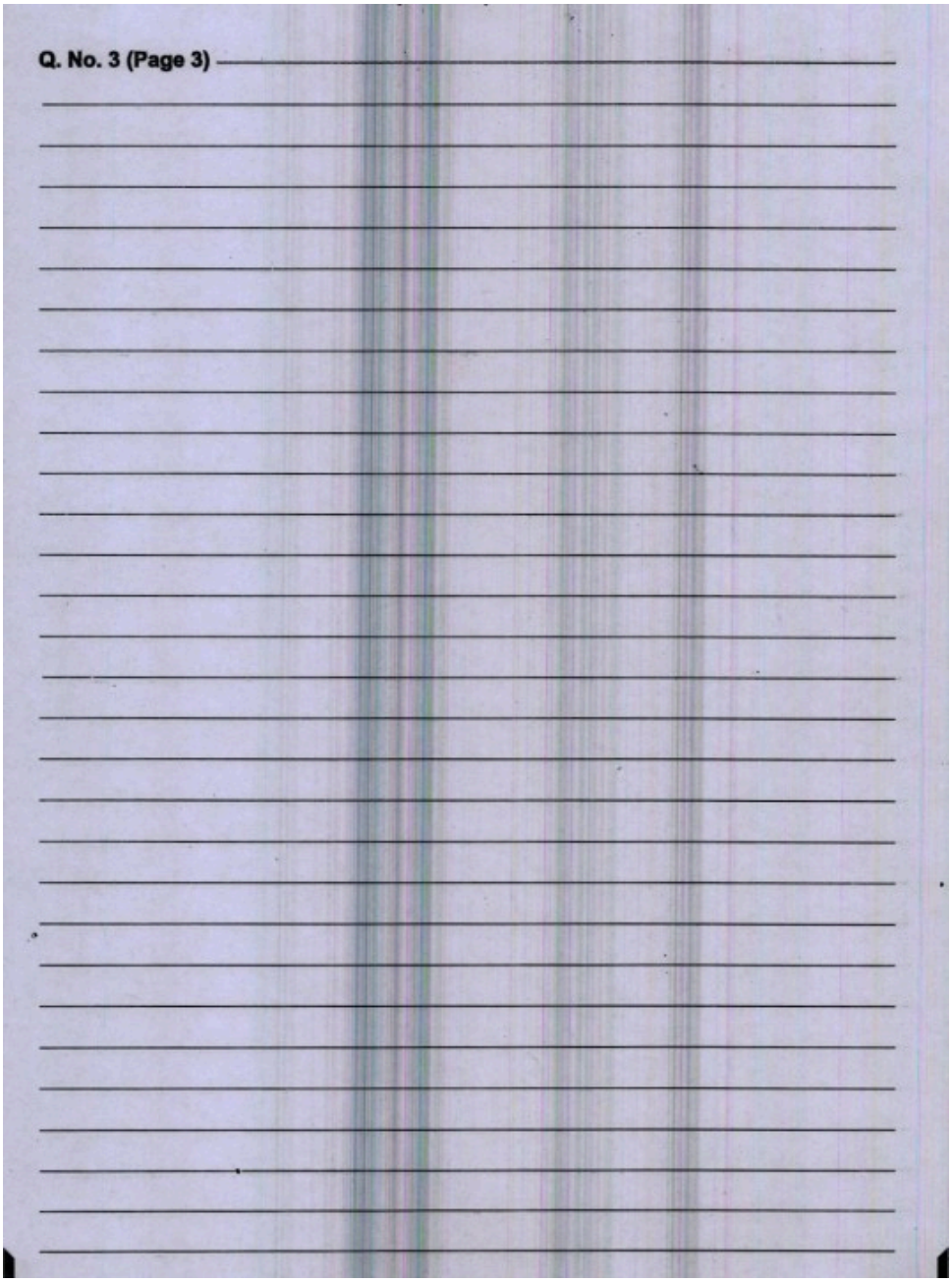
$$(M_{normal})(M_{tan}) = -1$$

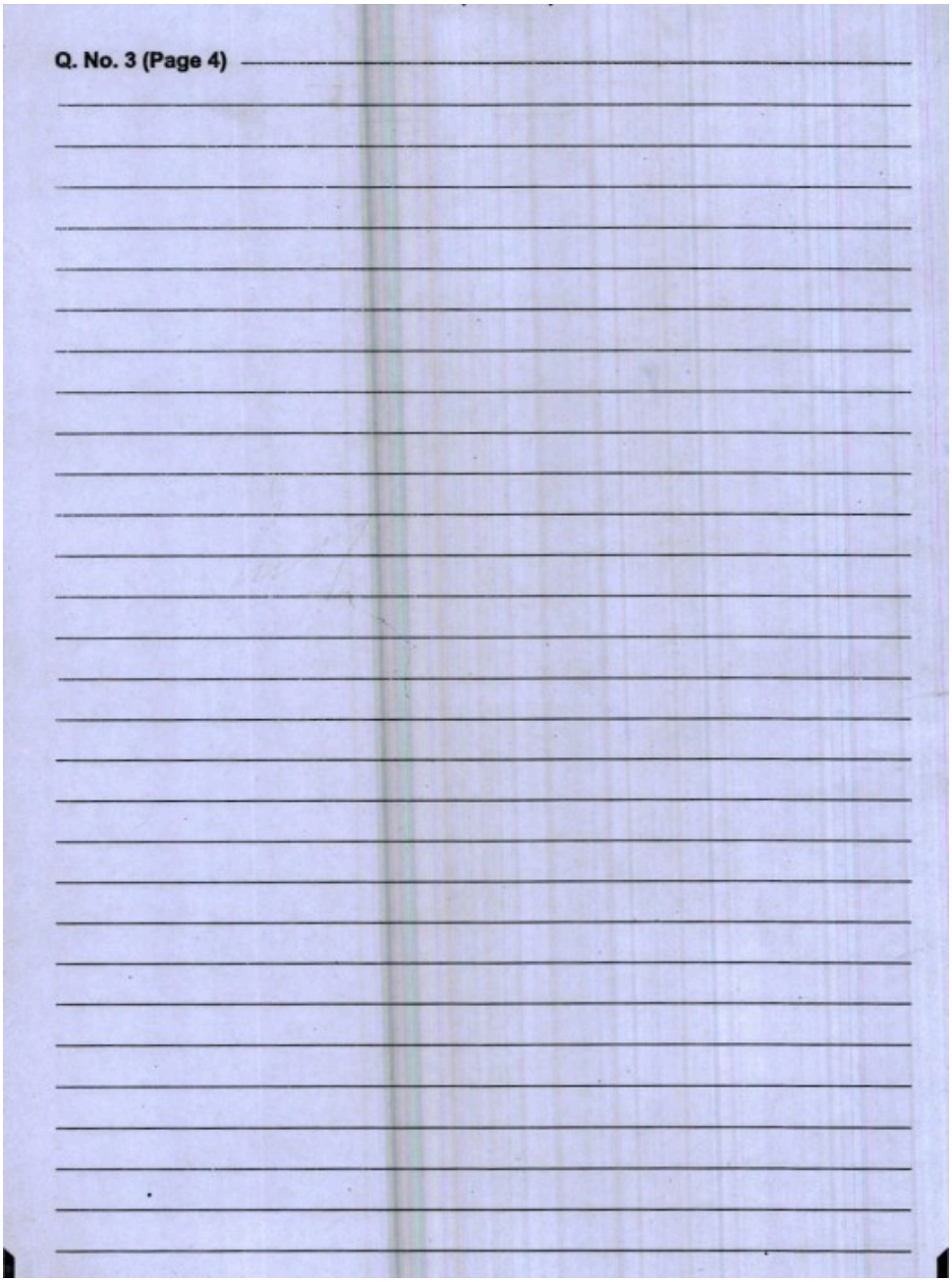
$$M_{normal} = \frac{-1}{M_{tan}} = \frac{-1}{1} = -1$$

$$y - 2 = -1(x - 0)$$

$$y - 2 = -x$$

$$\therefore x + y - 2 = 0$$


Q. No. 3 (Page 3)


Q. No. 3 (Page 4)

Q. No. 4 (Page 1) $y = \sin^{-1}(x)$ $x \in [-2\pi, 2\pi]$

Domain : $[-1, 1]$ graph cannot

Range : $[-\pi/2, \pi/2]$ be drawn for

Intercepts : $(0, 0)$, ~~$(0, 0)$~~ given domain so

End-points : $(-1, -1.6)$, $(1, 1.6)$ we use $[-1, 1]$

x	-1	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	1
y	-1.6	-0.8	-0.5	-0.3	0	0.3	0.5	0.8	1.6

x-intercept $[y = 0]$

$\sin^{-1}(x) = 0$

$x = \sin(0)$

$x = 0$

x-intercept : $(0, 0)$

y-intercept $[x = 0]$

$y = \sin^{-1}(0)$

$y = 0$

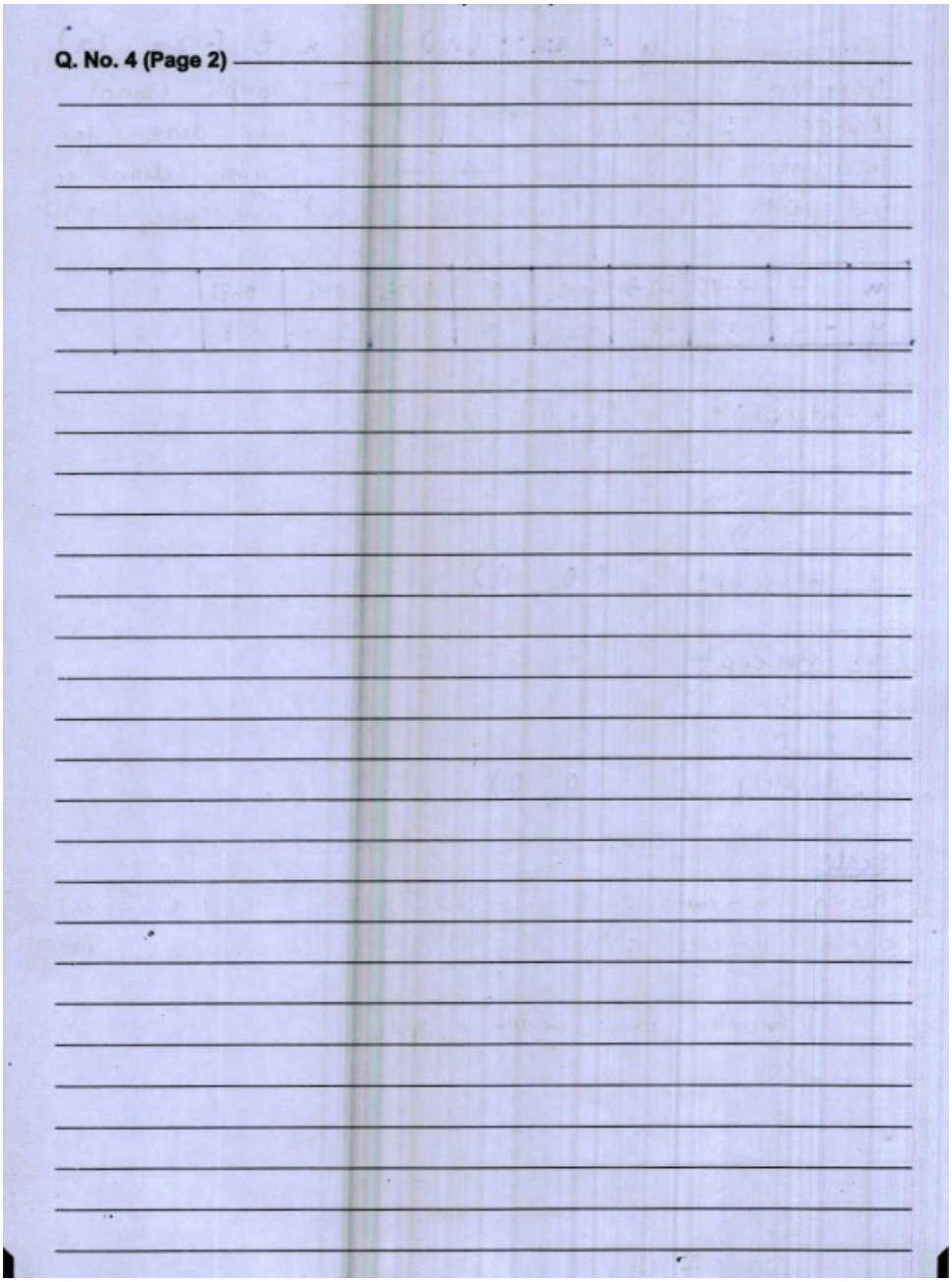
y-intercept : $(0, 0)$

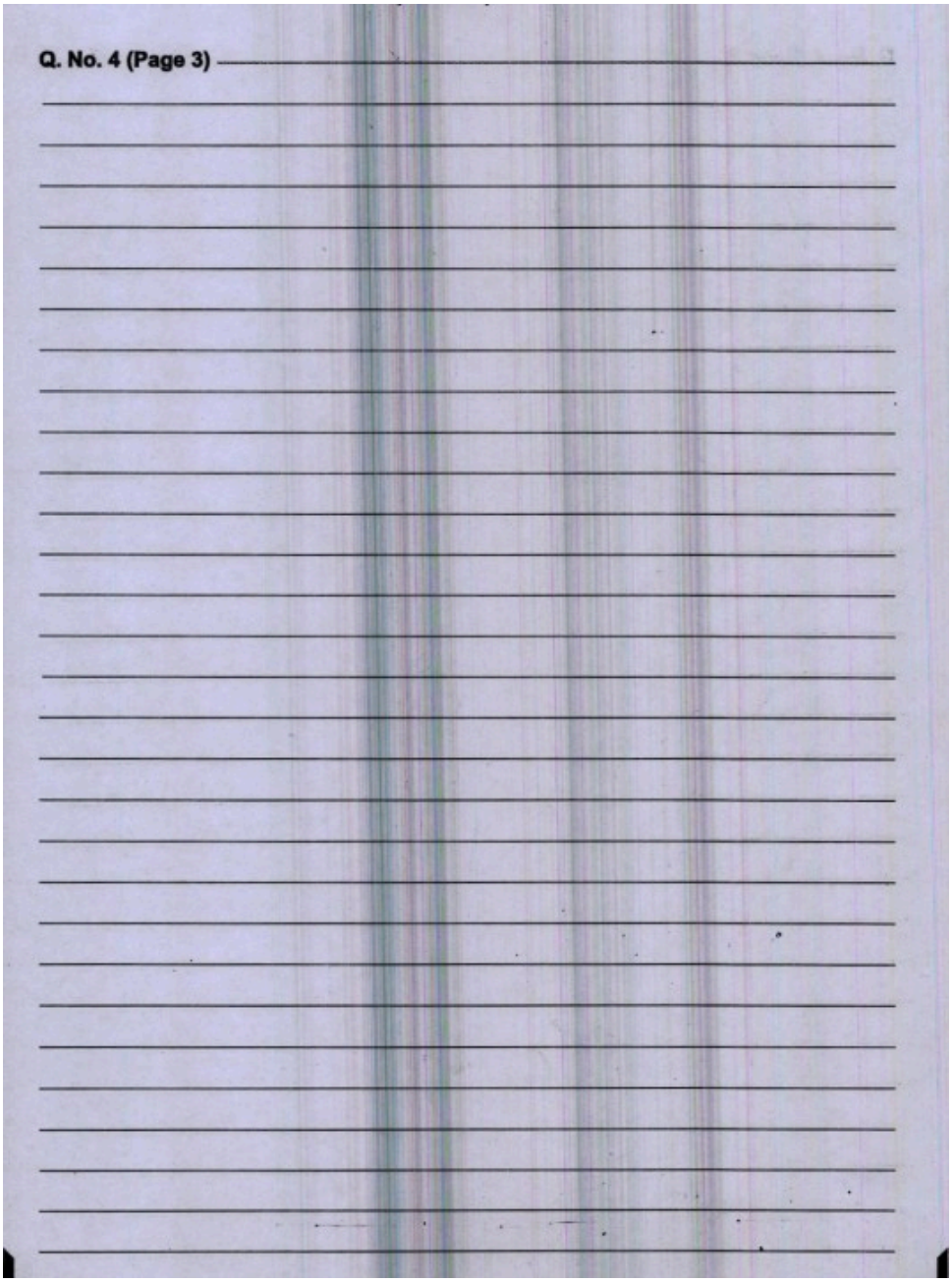
Scale

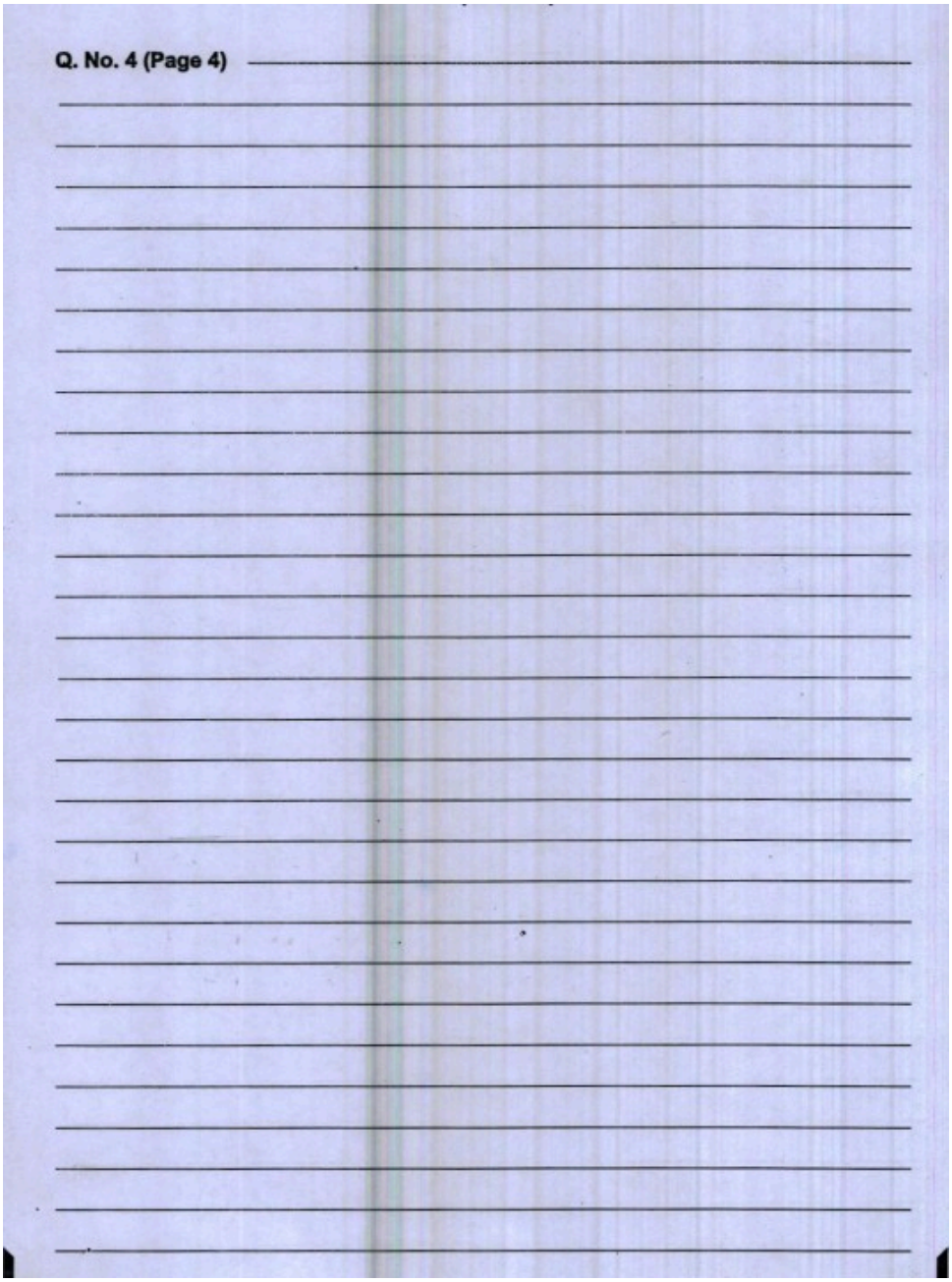
along x-axis : 1 big block = 0.5 (1 small block = 0.05)

along y-axis : 1 big block = 1 (1 small block = 0.1)

[Graph on graph paper]


Q. No. 4 (Page 2)


Q. No. 4 (Page 3)


Q. No. 4 (Page 4)

Q. No. 5 (Page 1) a. $\dfrac{x^2+3x+4}{(x^2+2x+5)(x-1)}$

$$\frac{x^2+3x+4}{(x^2+2x+5)(x-1)} = \frac{A}{(x-1)} + \frac{Bx+C}{(x^2+2x+5)}$$

Multiplying b.s by $(x^2+2x+5)(x-1)$

$$x^2+3x+4 = A(x^2+2x+5) + (Bx+C)(x-1)$$

for $x = 1$

$$1^2+3(1)+4 = A[1^2+2(1)+5] + (Bx+C)(1-1)$$

$$8 = A(8)$$

$$A = 8/8$$

$$A = 1$$

comparing coefficients of 'x^2'

$$1 = A + B$$

$$B = 1 - A$$

$$B = 1 - 1$$

$$B = 0$$

comparing coefficients of 'x'

$$3 = 2A - B + C$$

$$3 = 2(1) - 0 + C$$

$$3 = 2 + C$$

$$C = 3 - 2$$

$$C = 1$$

$$\therefore \frac{x^2+3x+4}{(x^2+2x+5)(x-1)} = \frac{1}{(x-1)} + \frac{1}{(x^2+2x+5)}$$

Q. No. 5 (Page 2)

b. $\int \frac{u^2+3u+4}{(u^2+2u+5)(u-1)}\,du = \int \frac{1}{(u-1)}\,d + \frac{1}{u^2+2u+5}\,du$

$= \int \frac{1}{(u-1)}\,du + \int \frac{1}{u^2+2u+5}\,du$

$\int \frac{1}{u^2+2u+5} = \int \frac{1}{(u)^2+2(u)(1)+1^2-1^2+5}$

$= \frac{1}{(u+1)^2+(4)} = \frac{1}{(u+1)^2+2^2}$

$\int \frac{1}{(u+1)^2+2^2}\,du$

let $x = u+1 = 2\tan\theta \qquad \theta = \tan^{-1}\left(\frac{u+1}{2}\right)$

$\frac{d(u+1)}{d\theta} = 2\sec^2\theta$

$du = 2\sec^2\theta\,d\theta$

$= \int \frac{1}{(u+1)^2+2^2} = \int \frac{2\sec^2\theta}{(2\tan\theta)^2+2^2}\,d\theta$

$= \int \frac{2\sec^2\theta}{4(\tan^2\theta+1)}\,d\theta = \int \frac{\sec^2\theta}{2(\sec^2\theta)}\,d\theta$

$= \frac{1}{2}\int d\theta = \frac{1}{2}(\theta) + c$

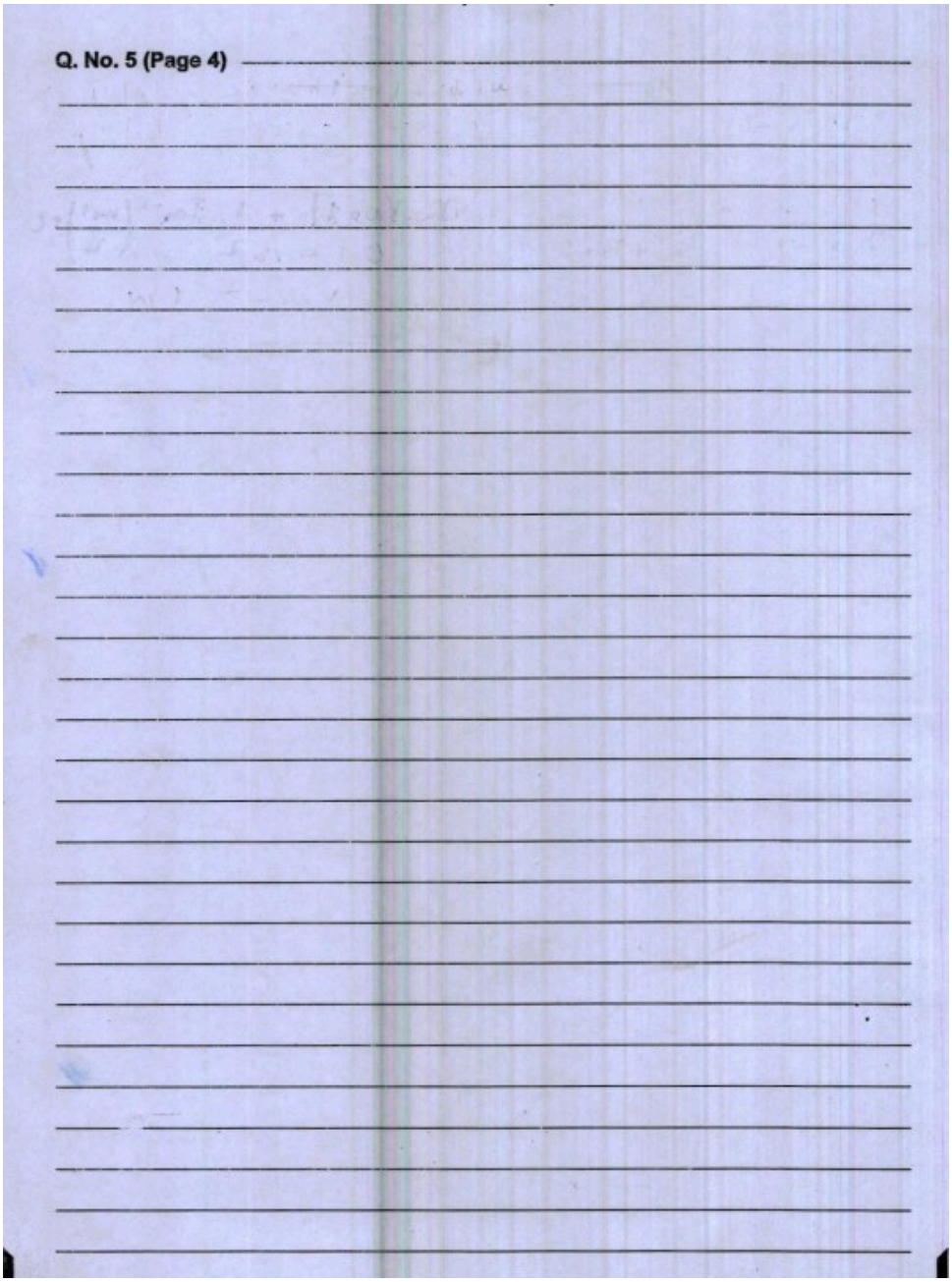
Putting value of θ

$= \frac{1}{2}\tan^{-1}\left(\frac{u+1}{2}\right) + c$

Q. No. 5 (Page 3)

$$\int \frac{1}{(x-1)}\,dx = \ln|x+1| + c$$

$$\int \frac{1}{x-1} + \frac{1}{x^2+2x+5} = \ln|x+1| + \frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right) + c$$


Q. No. 5 (Page 4)

Q. No. 6 (Page 1) $\frac{dy}{dx} = \frac{x^2+y^2}{xy}$

Let $y = ax$

$$\frac{dy}{dx} = a(1) + x\left(\frac{da}{dx}\right)$$

$$\frac{dy}{dx} = a + x\frac{da}{dx}$$

$$\frac{x^2 + (ax)^2}{x(ax)} = a + x\frac{da}{dx}$$

$$\frac{x^2(1+a^2)}{x^2 a} = a + x\frac{da}{dx}$$

$$\frac{1+a^2}{a} - a = x\frac{da}{dx}$$

$$\frac{1+a^2-a^2}{a} = x\frac{da}{dx}$$

$$\frac{1}{a} = x\frac{da}{dx}$$

$$\frac{1}{x}\,dx = a\,da$$

integrating b.s

$$\int \frac{1}{x}\,dx = \int a\,da$$

$$\ln|x| = \frac{a^2}{2} + c$$

Putting value of a

$$\ln|x| = \frac{(y/x)^2}{2} + c$$

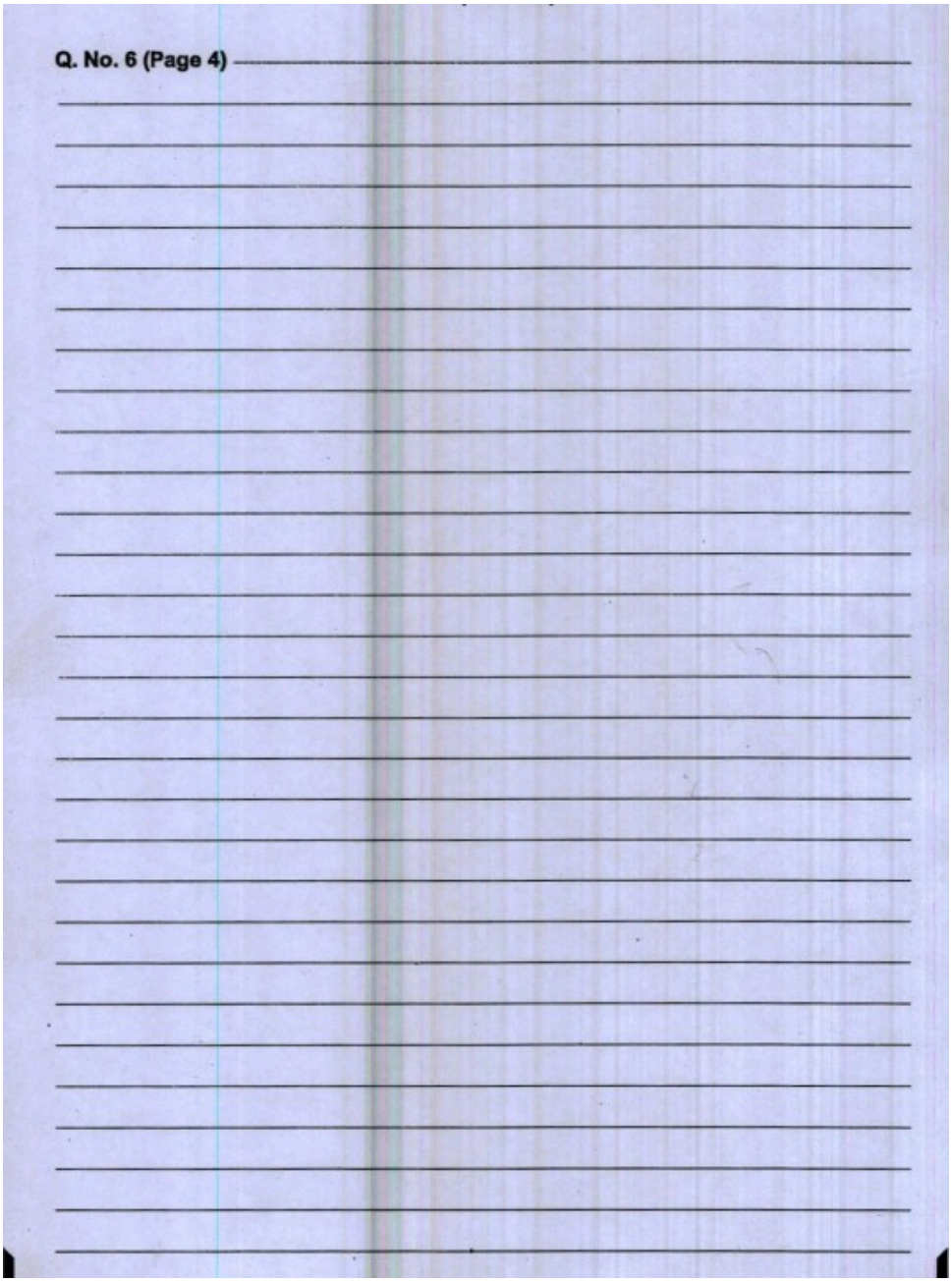
$$\ln|x| = \left(\frac{y^2}{x^2}\right)\left(\frac{1}{2}\right) + c$$

Q. No. 6 (Page 2)

$$\ln|u| = \frac{y^2}{2u^2} + C$$

$$2u^2(\ln|u|) - y^2 = C$$

Q. No. 6 (Page 3)


Q. No. 6 (Page 4)